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Can Supply Shocks Be Infaltionary with a Flat Phillips Curve?

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Abstract

Empirical estimates find that the relationship between inflation and the output gap is close to nonexistent—a so-called flat Phillips curve. We show that standard pricing frictions cannot simultaneously produce a flat Phillips curve and meaningful inflation from plausible supply shocks. This is because imposing a flat Phillips curve immediately implies that the price level is *also* rigid with respect to supply shocks. In quantitative versions of the New Keynesian model, price markup shocks need to be several orders of magnitude bigger than other shocks in order to fit the data, leading to unreasonable assessments of the magnitude of the increase in costs during inflationary episodes. Hence, we propose a strategic microfoundation of price stickiness in which prices are sticky with respect to demand shocks but flexible with respect to supply shocks. In our model, the friction leading to rigidities is demand-intrinsic, in line with narrative accounts for the imperfect adjustment of prices. Firms can credibly justify a price increase due to a rise in costs, whereas it is harder to do so when demand increases. This has natural implications for inflation dynamics and crucial implications for the conduct of monetary policy.

Keywords: Cost-push shocks, customer markets, average-cost pricing, price stickiness.
JEL classification: E31, E52, E58

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1 Introduction

The current global rise in inflation presents a challenge for macroeconomics. For over three decades, inflation was virtually a non-story in advanced economies. Inflation remained incredibly stable, even amidst several large and global recessions and substantial changes in fiscal and monetary policy. One could say that there was a "great moderation" in inflation even as output remained anything but moderate at times. In other words, the relationship between inflation and demand—the so-called *Phillips curve* (PC)—has been flat, suggesting that demand shocks are not important determinants of variations in inflation.

And yet, inflation globally is now running at levels not seen in decades. In fact, such rates of inflation have not been observed in developed countries since the Great Inflation of the 1970s. Moreover, the conjunction of higher inflation with global supply factors, both in the 1970s and in the current episode, is hard to overlook. Indeed, it suggests that supply is a major determinant of inflation.[1]

Thus, our paper is motivated by two facts regarding the dynamics of inflation.

Fact 1: The Phillips Curve Is Very Flat. There is a significant body of evidence that for advanced economies the PC is incredibly flat, with a slope that is small or close to zero. Hazell, Herreño, Nakamura, and Steinsson (2022) use cross-sectional data to provide evidence of only a modest flattening of the PC since 1990. According to their findings, the PC has *always* been incredibly flat. Del Negro, Lenza, Primiceri, and Tambalotti (2020) find overwhelming evidence in favor of a very flat PC, especially since 1990. Their findings are consistent with other New Keynesian medium-scale DSGE estimations. Both papers estimate slopes on the order of 0.002.[2]

Fact 2: Supply Shocks Are Inflationary. This claim is corroborated by mounting evidence that supply-driven sources explain a significant portion of the current inflation. Känzig (2021) finds that oil news shocks alone explain 50% of the forecast error variance decomposition in the U.S. Consumer Price Index (CPI), which provides a lower bound on how much supply disruptions affect inflation. In considering firm-level pricing decisions in the UK following the pandemic, Bunn,

[1] It is of course plausible that demand factors are *also* a major part of the explanation for the recent rise in inflation, especially in the U.S., which enacted a significant post-COVID fiscal package. However, demand alone does not readily explain inflation as a *global* phenomenon, or its magnitude and cross-country synchronization, especially given flat estimates of the Phillips curve.

[2] To be clear, the arguments in our paper do not necessarily rely on the PC being *very* flat. A *moderately* flat PC leads to the same remarks. We are more explicit about this in Section 2.

Anayi, Bloom, Mizen, Thwaites, and Yotzov (2022) conclude that "it is supply side factors that can explain most of the rise in inflation since 2021." They especially note the role of labor and materials shortages. Similarly, Ball, Leigh, and Mishra (2022) find that energy prices, supply-chain backlogs, and auto-related prices have been most important in explaining U.S. inflation. Papers with more modest findings attribute 40%–45% of the recent inflation to supply shocks (Di Giovanni et al., 2022; Shapiro, 2022).

Our first goal in this paper is to stress that these two facts represent a challenge for standard models aiming to account for short-run inflation dynamics. Consider first the New-Keynesian (NK) family of models. In these models, because of the flatness of the PC, the degree of price stickiness required to fit the data is very high (Del Negro et al., 2020; Hazell et al., 2022). The "Calvo fairy" does not visit firms very frequently. But then, supply shocks of a reasonable size cannot be inflationary, since the fairy *also* needs to visit firms for them to adjust prices when inflationary cost-push shocks hit.

A common misconception is that a flat PC means merely that variations in *demand* do not cause variations in inflation—whereas variations in *supply* could cause inflation. In reality, there are no empirically plausible structural shocks that will create significant variations in inflation when the PC slope is close to zero. NK models cannot deliver periods of high inflation volatility if the PC is flat, unless one is willing to accept the existence of cost shocks several orders of magnitude larger than other structural shocks. It goes without saying that at times, inflation volatility is far from zero. Using a simple calibration based on the estimates by Del Negro, Lenza, Primiceri, and Tambalotti (2020), we find that in order to generate a 1% increase in inflation, firms' desired markups need to increase by 500%. A steeper PC reduces this required markup increase somewhat, but actually not to reasonable levels. This observation is a variation on the critique in Chari, Kehoe, and McGrattan (2009), but here we show that the flat PC is the fundamental cause of the implausible price markup shocks that they highlight.

We also show that this point is quite general and does not only involve the NK model. A theoretical analysis explains that standard pricing frictions—such as Calvo, Taylor, menu, or Rotemberg costs—all suffer from the same caveat. The reason is the common feature of treating rigidity with respect to supply and demand symmetrically. Therefore, standard pricing frictions cannot account for significant variations in inflation (such as the recent episode), *and* at the same time predict a flat Phillips curve, which is clearly evidenced by the missing disinflations during the Great Recession and the onset of the COVID recession, plus the "missing inflations" during the housing boom of the early 2000s, and during the unprecedented expansion of the Federal Reserve's balance sheet of the last decade.[3]

Our second goal, therefore, is to present a model that can address the Facts 1 and 2. As we observed, the data asks for a model that treats demand and supply disturbances *asymmetrically*. We provide a microfounded model of price stickiness in which inflation can be entirely rigid with respect to demand shocks and yet entirely flexible with respect to supply shocks. Our model can simultaneously produce a very flat PC while also producing significant inflation in response to supply disturbances.

The proposed microfoundation is based on strategic price-setting decisions of firms. Our model captures a realistic firm-consumer interaction where firms are concerned about consumers' reaction to price changes, building on the models of L'Huillier (2020) and L'Huillier and Zame (2022). Consumers have less information than firms, and they do not want to be misled or taken advantage of by better-informed firms. In this environment, sticky prices can arise when the asymmetry of information is severe. The model uses standard game theory tools to handle this strategic interaction, and we do not make any behavioral or nonstandard assumptions.[4]

In this setting, when the volatility of residual demand shocks is moderate (i.e., after accounting for the endogenous response of policy in offsetting fundamental shocks), many firms find it optimal to post sticky prices. It can even be the case that all firms in the economy choose prices that do not vary with demand at all. Thus, our microfoundation can produce a PC that is flat, even with slope zero, matching the empirical evidence in Fact 1.

Importantly, our model of price stickiness has novel predictions for the effects of shocks to the supply side of the economy that are radically different from the predictions obtained with other price stickiness models. In particular, while prices can be completely sticky with respect to demand shocks, prices remain completely flexible with respect to supply shocks, thus addressing Fact 2.

The intuition for the asymmetry in the adjustment of prices depending on the type of shock is the following. When demand increases, *uninformed* consumers are concerned that an increase in prices merely reflects an undue increase in profits (when, in fact, the increase in prices would be justified

[3] Of all these examples, perhaps the most telling is the onset of COVID in Q2 2020. The U.S. real GDP fell by more than 32% (annualized) and unemployment rose to 15%, but headline CPI inflation only fell by less than 4%, to rise again in Q3 2020.

[4] L'Huillier (2020) shows that this model delivers hump-shaped dynamic responses of both output and inflation, even in the absence of bells and whistles. Thus, these models also deliver realistic predictions for the propagation of shocks. L'Huillier and Zame (2022) show that the price stickiness result is robust to the consideration of optimal mechanisms and contract setting.

by the increase in demand). Since the firm would always like to pretend there is high demand for its product, it is unable, in a Perfect Bayesian equilibrium, to credibly convince uninformed consumers that it is rightly increasing the price. This strategic situation leads, optimally, to price stickiness, as shown in L'Huillier (2020). The situation is quite different when costs increase. In that case, a firm can credibly pass on the cost increase to all consumers (informed and uninformed), since it does not lead to an increase in profits. In fact, the firm could be losing revenue due to the price increase, but it would be optimally reacting to the cost increase. Therefore, there is no strategic consideration involved when cost shocks hit because uninformed consumers do not worry that the firm is cheating by raising prices in response to cost shocks. In a nutshell, demand shocks lead to strategic considerations for pricing, while supply shocks do not.[5] This explains the asymmetric response of prices.[6]

Thus, our model can simultaneously produce a flat Phillips curve and large responses of inflation to supply disturbances. This result is more than a theoretical curiosity; it helps rationalize the recent empirical evidence by Bunn et al. (2022), Känzig (2021) and Ball et al. (2022), among others, which favors the interpretation that the supply shocks are primarily responsible for the recent inflation. This is precisely what our model predicts.

The fact that nominal rigidities are exclusively demand-intrinsic in our model offers thought-provoking results regarding the optimal behavior of a central bank facing supply-driven inflation. Following a supply disruption, a rise in the price level is socially optimal and simply reflects firms' lower productivity or higher costs of production. Higher prices result in lower demand, which is efficient. Therefore, there is no justification for central bank action because there are no welfare losses due to price rigidities. If the central bank raises interest rates to lower inflation, it will generate a negative demand shock. The price level will turn sticky downward, and an inefficient negative output gap will result as a consequence of the extra fall in demand generated by the central bank. In our model, price level fluctuations due to shifts in the supply side of the economy are

[5] As a matter of fact, our model is one where firms would prefer to communicate that price increases are due to cost increases ("we are raising our prices because our costs have increased"), rather than demand increases ("we are raising our prices because our product has become more popular"). Whereas this is not exactly how firms behave in the model, it is a useful thought to get intuition.

[6] Formally, there are separating Perfect Bayesian Equilibria (flexible prices) and pooling Perfect Bayesian Equilibria (sticky prices). In the case of demand shocks, Perfect Bayesian Equilibria generate an endogenous cost to price adjustment due to the inability of the firm to commit to truthfully revealing the shock. When there are many uninformed consumers, the firm is better off posting a sticky price. In the case of supply shocks, there is no cost to price adjustment since the firm does not have an incentive to misrepresent the shock (if costs are low, the firm prefers lowering, not raising, prices). If we model a markup shock (following the standard treatment as in the NK model), our model, again, generates price flexibility. The reason is that shifts in desired markup shocks can only originate in the contemporaneous demand function of consumers, and hence they are common knowledge.

optimal and should not be actively stabilized. This type of prediction highlights the usefulness of microfounding the strategic sources of the price friction. Our paper underlines the importance of capturing the asymmetric response of inflation commanded by the data when thinking about the optimal monetary response to alternative shocks.

Related Literature. There is a classic literature providing evidence that the firm-customer relation is what limits price adjustment, suggesting that nominal price-setting frictions are demand-based (Hall and Hitch, 1939; Okun, 1981; Kahneman et al., 1986; Greenwald and Stiglitz, 1989; Blinder, 1991). Blinder et al. (1998) provide survey evidence that when asked to explain their reluctance to increase prices after an increase in costs, firms' managers usually answer that "price increases cause difficulties with customers." For modern evidence, see also Rotemberg (2005) and Nakamura and Steinsson (2011).

There is a robust literature that attempts to reconcile the New Keynesian framework with the data. The first major challenge, or anomaly, is explaining (or reinterpreting) the so-called missing inflations during the Great Recession. Several factors have been considered to explain (or reinterpret) these phenomena, such as inflation expectations (Jorgensen and Lansing, 2019), online retail (Cavallo, 2018), and globalization (Forbes, 2019). See L'Huillier and Schoenle (2019) for related evidence of the link between the frequency of price adjustment and the inflation target.

Motivated by the previous findings, a burgeoning literature studies the shape of the Phillips curve, finding evidence of non-linearities and a flattening slope (Ascari and Fosso, 2021; Ascari et al., 2022; Harding et al., 2022a,b). Benigno and Ricci (2011) shows that downward nominal wage rigidities produce nonlinear PCs that flatten at low inflation. Coibion, Gorodnichenko, and Kamdar (2018) consider how belief formations affect the PC. McLeay and Tenreyro (2020) show that, away from the zero lower bound, the combination of cost-push and demand shocks leads to difficulties with identification of the slope of the Phillips curve.

Many papers recognize that different microfoundations for price stickiness have important implications for inflation and monetary policy (Ascari, 2004; Caballero and Engel, 2007; Karadi et al., 2022; Ascari and Haber, 2022) and that the Phillips curve is endogenous, with important implications (Kiley, 2000; Levin and Yun, 2007; Kocherlakota, 2021; Gaballo and Paciello, 2021; Petrosky-Nadeau and Bundick, 2021). Werning (2022) shows that the effect of inflation expectations on aggregate inflation depends on the microfoundation for price stickiness.

There is a robust literature studying information frictions in price setting, pioneered by Mankiw and Reis (2002). Ball, Mankiw, and Reis (2005) show that when price setters are slow to incorporate macroeconomic information into the prices they set, the optimal monetary policy is price level targeting. Acharya (2017) considers a sticky information model in which the endogenous decision of when to acquire new information about different shocks leads prices to change frequently and by large amounts in response to idiosyncratic shocks but sluggishly in response to monetary shocks. Gutiérrez-Daza (2022) considers an economy in which consumers learn from shopping. Bernstein and Kamdar (2022) study optimal monetary policy with central bank inattention.

2 New Keynesian Model: No Cost-Push Inflation with a Flat Phillips Curve

The New Keynesian (NK) Phillips curve is generally written as

$$\widehat{\pi}_t = \beta \mathbb{E}_t[\widehat{\pi}_{t+1}] + \kappa \widehat{x}_t + \lambda \widehat{z}_t, \tag{1}$$

where $\widehat{x}_t$ denotes the output gap, $\widehat{z}_t$ denotes a *structural* cost-push shock, and κ and λ are parameters: κ measures the sensitivity of inflation to demand fluctuations in equilibrium, while λ *by definition* measures the sensitivity of inflation to supply shocks. The parameter κ is what is commonly called the Phillips curve slope.

The theme we develop in this section is that empirical estimates of κ are very small, and therefore, variations in $\widehat{x}_t$ are unlikely to create inflation. Instead, inflation must come from cost-push shocks $\widehat{z}_t$. But, the NK model predicts symmetric sensitivities. The parameters κ and λ are proportional to each other. If κ is very small, so must λ, for any plausible proportionality factor. For the model to generate variation in inflation through shocks to $\widehat{z}_t$, the parameter λ cannot be too small. In sum, the empirical facts suggest that κ, and therefore λ, are very low—in fact, close to zero—and yet there are shocks that can create meaningful inflation. This generates a tension within the NK model. In practice, this results in unreasonably large shocks $\widehat{z}$ to fit the data. The resulting shocks are so large that they can hardly be considered microfounded desired changes in markups or costs.

To understand our argument, recall that the standard NK PC can equivalently be written in linearized form as

$$\widehat{\pi}_t = \beta \mathbb{E}_t[\widehat{\pi}_{t+1}] + \lambda \widehat{mc}_t, \tag{2}$$
$$= \lambda \sum_{s=0}^{\infty} \beta^s \mathbb{E}_t[\widehat{mc}_{t+s}],$$

where $\widehat{mc}_t$ is some measure of deviations in firms' marginal costs, β is the discount factor, and λ is a function of structural parameters (the degree of price stickiness, the elasticity of substitution across goods, and the capital share in production). The parameter λ measures how changes in marginal costs translate into changes in prices. The second equality follows by iterating forward. Hence, within the standard NK setting, inflation is caused by changes in firms' marginal costs.

Marginal costs generally increase because output gaps increase (there is curvature in production and labor supply) or costs increase for the same level of production. In addition, firms may have a higher desired price for a given marginal cost, typically modeled as a shock to desired markups, which is generally called a *cost-push shock*. When marginal costs increase due to an output gap, the Phillips curve is most often written in the following conventional form

$$\widehat{\pi}_t = \beta \mathbb{E}_t[\widehat{\pi}_{t+1}] + \kappa \widehat{x}_t,$$

where $\widehat{x}_t \equiv \left(\sigma + \frac{\varphi+\zeta}{1-\zeta}\right)^{-1} \widehat{mc}_t$ is the output gap, $\kappa \equiv \lambda \left(\sigma + \frac{\varphi+\zeta}{1-\zeta}\right)$ is the PC slope, σ is the elasticity of intertemporal substitution (EIS), φ is the inverse Frisch elasticity, and ζ is the curvature of the production function. For the plausible values of $\sigma = 1$, $\varphi = 5$, and $\zeta = 1/3$, then $\left(\sigma + \frac{\varphi+\zeta}{1-\zeta}\right) = 9$, implying that λ is an order of magnitude smaller than κ in the standard NK model. In the NK model, one cannot have a very low κ without a very low λ.[7]

The Phillips curve is estimated to be very flat. Hazell et al. (2022) estimate the slope of the Phillips curve in the cross section of U.S. states and find $\kappa = 0.0062$ using their preferred specification that uses an IV for unemployment. Using an IV for tradable demand, they estimate $\kappa = 0.0020$. These estimates point to incredibly low levels of price flexibility.[8] Del Negro et al. (2020) also find considerable levels of rigidity. They use a DSGE model to estimate the coefficient multiplying marginal costs directly. Their mean posterior estimates of λ are 0.015 pre-1990 and

[7] In the standard model, λ is a function of the index of price stickiness, the elasticity of substitution across goods, and the production function curvature. As is well-known, the micro-level estimates of these parameters imply a λ that is orders of magnitude larger than the estimated value based on carefully identified empirical research.

[8] This estimate for κ at 0.0020 is taken from Table C.2 and based on their calibration. Introducing the preference calibration from Galí (2015) would deliver a value for κ 9 times smaller.

0.0015 post-1990. The empirical estimates therefore imply a value of λ that is no more than 0.015 in general and as small as 0.0015 post-1990.

More generally, if firms' desired prices can also change for reasons independent of the output gap (e.g., markup shocks), then we can write the NK PC in a similar form, as in equation (1). Defining a the reduced-form shock $\nu_t \equiv \lambda \widehat{z}_t$, we write

$$\widehat{\pi}_t = \beta \mathbb{E}_t[\widehat{\pi}_{t+1}] + \kappa \widehat{x}_t + \nu_t,$$

where $\nu_t \equiv \lambda \widehat{z}_t$ simply redefines the cost shock and $\widehat{z}_t$ represents the *structural* cost-push shock. While variations in $\widehat{x}_t$ cannot create inflation because κ is so small, variations in the *reduced-form* shock ν_t, which affects inflation one-for-one, can.

This last step is problematic when it comes to empirics. Matching the empirical behavior of inflation requires ascribing a great deal of volatility to the ν_t term. But this is a *reduced-form* shock. The structural shock is the change in underlying costs $\widehat{z}_t$. If $\lambda = 0.002$ (rounding up the post-1990 mean posterior estimate from Del Negro, Lenza, Primiceri, and Tambalotti 2020 at 0.0015), then generating a 1% variation in ν_t requires a change in $\widehat{z}_t$ by $1\%/0.002 = 500\%$. If steady-state markups are 12.5%, then a shock of this size requires that markups increase to 75.0%, an implausible change at business cycle frequency. A 1% increase in inflation due to a cost-push shock requires implausibly large structural shocks. The standard normalization is far from innocuous.

Using the estimates from Hazell, Herreño, Nakamura, and Steinsson (2022) yields similar conclusions. If $\lambda = 0.0062$, then generating a 1% variation in ν_t requires a change in $\widehat{z}_t$ by $1/0.0062 = 161\%$. If steady-state markups are 12.5%, then a shock of this size requires that markups increase to 32.7%. Clearly, this is, again, an implausible change at business cycle frequency. Also, using their estimate based on a tradable-demand IV of $\lambda = 0.0020$ delivers the same conclusions as the rounded-up value for Del Negro et al. (2020) used above.

How do these observations square with the recent global rise in inflation since the second quarter of 2021? Importantly, this rise in inflation has been much larger than 1%. For the sake of the argument, consider an increase in costs capable of rising inflation, from its steady state value, by 5 percentage points (say from 2% to 7%). To generate such an increase with a value of $\lambda = 0.002$ ($\lambda = 0.0062$) requires a change in $\widehat{z}_t$ of 2500% (806%). In this scenario, markups increase from 12.5% to 325% (113.3%). Both in the 1970s and in the recent episode, the effect of supply shocks on inflation has been large. Therefore, the observed large increases in inflation aggravate the issue raised by the usual normalization of cost-push shocks in the NK model.

This previous analysis implies that there are no plausible shocks—whether driven by demand or supply—that can create meaningful inflation. Some shocks that we typically associate with supply, such as productivity shocks, show up in the NK model within $\widehat{x}_t$, as a shock to the output gap. Depending on how shocks are motivated (e.g., as efficient changes in labor demand or as inefficient changes in wage-markups), different structural shocks can show up in variations in $\widehat{x}_t$ or in $\widehat{z}_t$, or in some combination. But if κ and λ are both approximately zero, then no structural shocks can generate inflation. Shocks that primarily show up in the output gap cannot generate inflation because κ is close to zero. Shocks that primarily show up as cost-push shocks varying $\widehat{z}_t$ cannot generate inflation because λ is close to zero.

In Sections 3 and 4, we argue that this observation suggests considering price-setting frictions with asymmetric price rigidities with respect to supply and demand disturbances. In essence, we propose a model in which λ and κ are not directly related and one can have a very low κ without a very low λ.

The normalization $\nu_t \equiv \lambda \widehat{z}_t$ is problematic conceptually for the reasons we just gave. However, over time this problem has become more significant as the Phillips curve appears to have flattened (Del Negro et al., 2020; Hazell et al., 2022) over the years. Some commentators have suggested that the very recent inflation episode is evidence that the Phillips curve has steepened again. We are very cautious about overturning careful empirical work based on decades of data and thoughtful identification strategies on the basis of one episode of high inflation. However, even if the slope has increased, say, ten-fold, the problem remains: based on our first calculation above, structural shocks would need to be at least 50% to generate a 1% reduced-form shock, or a 250% shock to generate a 5% increase in inflation.

There are laudable contemporaneous attempts within the literature to improve the ability of the NK model to match inflation dynamics, for example by taking seriously non-linearities or how belief formation affects the PC (e.g., Ascari and Fosso, 2021; Ascari et al., 2022; Harding et al., 2022a,b). This literature moves the NK model in the right direction because it reconciles the observed flat PC with the rise in inflation observed after 2021:Q2. In this vein, our contribution is targeted at de-linking the tight restriction that symmetric rigidities impose within the NK model. We argue that incorporating asymmetric price rigidities with respect to supply and demand disturbances will provide macroeconomists with the ability to match the empirical facts noted in the introduction.

3 A Simple Model with Price-setting Frictions

In this section, we generalize the issue that arises within the NK model to a wider range of standard pricing frictions. We write a simple model to make the point that fixed costs, quadratic costs, and Calvo-Taylor, are all unable to account for cost-push inflation when the PC is flat. We explain that this is due to the symmetry embedded in these models regarding the reasons for the stickiness.

For a coherent presentation, we set up a simple framework that allows us to handle fixed costs, quadratic costs, Calvo-Taylor, and the strategic microfoundation used in Section 4. To this end, we need a framework that guarantees enough tractability. With this in mind, the framework we use is parallel to L'Huillier (2020), with the addition of shocks to supply. For ease of exposition, we maintain a simple two-period, partial-equilibrium model. The same points could be made in a much more complicated infinite-horizon, general-equilibrium model, which is relegated to Appendix B.

3.1 Model Setup

There are two dates, the present and the future, which we interpret as the short run and the long run. In the short run, there is production and trading in goods markets will be subject to frictions; in the long run, agents have exogenous endowments and trading will be frictionless. All that follows is common knowledge.

Setup: Agents and Markets. The economy is populated by firms, consumers, and a central bank (CB). At each date, firms and consumers trade in a market for a single good. Short-run markets are decentralized; we formalize this by positing a continuum of islands, each served by a single monopolistic firm and populated by a continuum of consumers. Long-run markets are centralized; we formalize this by positing that all consumers trade endowments in a perfectly competitive market.

For convenience, we follow Lagos and Wright (2005), and denote present variables in lowercase and future variables in uppercase. Thus, the good in the present is c and its price is p; the good in the future is C and its price is P. For simplicity, we normalize the long-run price to $P = 1$.

There is a short-run bond market with nominal interest rate i. We posit a cashless economy in which the CB sets the nominal interest rate i. In this partial equilibrium setup, there is no labor supply.[9]

[9] Appendix B presents an equivalent model with labor supply and monetary frictions where the CB sets money

Consumer Problem. We index a typical consumer by j. All consumers have the same quasilinear utility function $U(c, C) = (c - c^2/2) + \theta C$, where $\theta \equiv \frac{1}{1+\rho}$ is the discount factor with discount rate ρ. Consumers have a real endowment E in the future and receive firm profits d in the present.[10] Hence consumer j solves

$$\max_{c,C} \quad (c - c^2/2) + \theta C$$
$$\text{s.t. } pc + QC = d + QE,$$

where p is the nominal price the consumer faces in the short run for goods and $Q \equiv \frac{1}{1+i}$ is the nominal price for bonds. In equilibrium, the consumer's optimal choice in the present will always lead to positive consumption in the future.

Consumer j's optimal short-run demand will be

$$c^* = 1 - p\frac{\theta}{Q} \equiv 1 - p\frac{1}{\xi},$$

where

$$\frac{1}{\xi} \equiv \frac{\theta}{Q} = \frac{1+i}{1+\rho}$$

is the ratio of the consumers' and the market's discount rates. Note that when ξ is high, demand in the present is high, and thus ξ acts as a demand shifter. In this section, we simply assume that ξ follows a known distribution with support over $(0, \infty)$.

Aggregate State. For now, we suppose that the aggregate state captures two dimensions of the economy: aggregate demand pressure and aggregate supply pressure. We model aggregate demand pressure as determined by consumers' discount rate, denoted by ρ. This modeling device is meant as a proxy for the many possible reasons that "the present" would, all else equal, be a good or bad time for consumers to spend. We model aggregate supply pressure as determined by firms' marginal costs, which we specify below.[11] The aggregate state describes the level of demand *in the present* and the cost of production. Hence, demand in the present will be *high* when consumers' discount *rate* is *high*. We suppose the CB responds to the aggregate state by setting the nominal

supply.

[10] As in Lagos and Wright (2005), quasilinearity in future consumption eliminates wealth effects; quadratic utility in present consumption is a computational convenience.

[11] In this and the following sections, we use the terms "supply shocks" and "cost shocks" interchangeably.

interest rate i. For now, we suppose the aggregate state is known by all agents.

Crucially, what matters for consumers' demand is the *net*, or *residual*, demand shock ξ, not the particular value of ρ independent of i. The shock ξ captures changes in demand caused by the changes in subjective and market discount factors. Thus, the residual demand shock ξ could be small if the CB offsets the shock to the discount factor ρ.

Firm Problem. Each island is populated by a single monopolistic firm. We assume that firms produce the consumption good at a constant marginal cost $z\xi$ with $z < 1$. We refer to z as the real marginal cost, on the timing assumption that production costs (wages or intermediate goods) are paid at the end of the first period and therefore the price is the discounted value of the price level in the future (see Appendix B). Then the firm profit given price p is

$$\left(1 - \frac{p}{\xi}\right)(p - z\xi). \tag{3}$$

With flexible prices, the optimal price is

$$p^* = \frac{\xi(1+z)}{2},$$

and total demand is

$$x = 1 - \frac{\xi(1+z)}{2\xi} = \frac{1-z}{2}.$$

Note that with flexible prices, output is fixed regardless of the demand shifter ξ. This is the natural level of output. We summarize the flexible-price equilibrium in the following lemma:

Lemma 1 (Flexible Prices)**.** *With flexible prices, all firms set a price* $p^* = \frac{\xi(1+z)}{2}$*, and demand at each island is* $x^* = \frac{1-z}{2}$*.*

3.2 Pricing Frictions and the Phillips Curve

We now introduce pricing frictions (more below). With pricing frictions, output will generally deviate from x^* when there is a shock. We can define a meaningful Phillips curve because nominal rigidities will lead to fluctuations in output and prices together. Given prices and output at each island, we define the *average* prices $\bar{p}$ and output $\bar{x}$, and we define a baseline aggregate state ξ_0, with average price $p_0 = \xi_0(1+z)/2$ (the flexible price) and output $\bar{x}_0 = \frac{1-z}{2}$.

We define the Phillips curve to be the two points $(\bar{x}, \bar{p})$ and $(\bar{x}_0, p_0)$. We define the *slope* κ of the PC to be the ratio of the (average) price difference to the (average) demand difference:

$$\kappa = \frac{\bar{p} - p_0}{\bar{x} - \bar{x}_0}.$$

We define a *supply shock* in this setting as changes in real marginal costs z. At this point, we can think of variations in z as capturing any variations in marginal costs, such as changes in the prices of intermediate goods or energy (like oil), changes in wages driven either by shocks to labor supply or shocks to wage-bargaining, or changes in productivity.[12]

Let λ denote the effect of marginal costs on aggregate prices. The response of inflation to supply shocks is given by

$$\lambda \equiv \frac{\partial \bar{p}}{\partial z}.$$

Thus, we are interested in characterizing κ as well as λ. We now consider three standard pricing-setting frictions in this simple model. To preview the results, standard price-setting frictions (e.g., quadratic adjustment costs, Calvo or Taylor frictions, and fixed adjustment costs) cannot produce $\kappa = 0$ and $\lambda > 0$.

Quadratic Costs. Let the firm face quadratic costs to adjust prices to p given by $\frac{\varphi}{2}(p - p_0)^2$, so $p_0 = \frac{1-z}{2}$ is the base price. The firm maximizes

$$(p - z\xi)(1 - p/\xi) - \frac{\varphi}{2}(p - p_0)^2. \tag{4}$$

We can summarize equilibrium as follows:

Lemma 2 (Quadratic Costs)**.** *With quadratic adjustment costs for prices, all firms set a price*

$$p^* = \xi \frac{1 + z + \varphi p_0}{2 + \varphi \xi},$$

and demand at each island is

$$x = \frac{1 - z + \varphi(\xi - p_0)}{2 + \varphi \xi}.$$

The PC slope in this economy is given by $\kappa = \frac{2}{\varphi}$.

Note that now output responds to the demand shock ξ because of nominal rigidities. Thus, a

[12] Because we simply want to point out that in these models rigidity with respect to shifts in demand implies rigidity with respect to shift in supply, whether the change in z represents an efficient or inefficient variation is not critical for the analysis.

flat PC corresponds to a very high φ (i.e., a high cost of adjusting prices). Asymptotically, if we want $\kappa \to 0$, we have $\varphi \to \infty$. In this setting, we cannot get inflation in response to cost shocks with a flat PC. We calculate

$$\frac{\partial p^*}{\partial z} = \frac{\xi}{2 + \varphi \xi}.$$

Thus, letting $\varphi \to \infty$, we have $\lambda \to 0$.

If the PC is flat, a cost shock will not lead to a large change in prices. The intuition is simple: a flat PC corresponds to a high cost of changing prices. It's still costly to change prices, whether responding to demand or supply shocks, and so firms don't change prices much.

Calvo-Taylor. Now suppose a fraction $1 - \varepsilon$ of firms cannot change their price. Thus, a fraction ε set their price to p^*, and the rest keep p_0 (i.e., there are no strategical complementarities arising from monopolistic *competition* à la Dixit-Stiglitz). This setting could correspond to probabilistic price changes à la Calvo or staggered price setting à la Taylor. In either case, the fraction of firms that adjust their price (whether probabilistic or pre-determined) is ε. The average price and output are thus

$$\bar{p} = \varepsilon \xi (1 + z)/2 + (1 - \varepsilon) p_0, \quad \bar{x} = \varepsilon \frac{1 - z}{2} + (1 - \varepsilon) \left(1 - \frac{p_0}{\xi}\right).$$

Lemma 3 (Calvo-Taylor)**.** *With Calvo or Taylor frictions, the PC slope in this economy is given by*

$$\kappa = \frac{\varepsilon}{1 - \varepsilon} \xi. \tag{5}$$

Thus, a flat PC corresponds to a very low ε (i.e., a low probability of price adjustment). Asymptotically, if we want $\kappa \to 0$, we have $\varepsilon \to 0$. Again, we cannot get inflation in response to cost shocks with a flat PC. We calculate

$$\frac{\partial \bar{p}}{\partial z} = \frac{\varepsilon \xi}{2}.$$

Thus, letting $\varepsilon \to 0$, we have $\lambda \to 0$.

Thus, if the PC is flat due to Calvo- or Taylor-style frictions, a cost shock will not lead to a large change in prices. The intuition is simple: a flat PC means few firms have the opportunity to change prices at all. If almost no firms can change prices, whether responding to demand shocks or supply shocks, firms won't change prices much in response to a cost shock.

Fixed Costs. Suppose firms pay a fixed cost $k_i \in [\underline{k}, \overline{k}]$ to adjust prices, with costs distributed according to CDF F. Adjusting firms choose $p^* = \frac{\xi(1+z)}{2}$, whereas not adjusting yields demand $x = 1 - p_0/\xi$, for a profit $d_0 = (p_0 - z\xi)(1 - p_0/\xi) = p_0(1+z) - z\xi - p_0^2/\xi$. A firm will adjust prices whenever

$$d^* \geq d_0 \implies k_i \leq k^*(\xi) \equiv \xi(1-z)^2/4 - (p_0(1+z) - z\xi - p_0^2/\xi).$$

Thus, the fraction of firms adjusting will be $F(k^*)$.

Lemma 4 (Fixed Costs)**.** *With fixed costs of adjusting prices, the PC slope in this economy is given by*

$$\kappa = \frac{F(k^*(\xi))}{1 - F(k^*(\xi))} \left(\frac{\xi(\xi - \xi_0)(1+z)}{1 + z - \xi_0} \right). \tag{6}$$

In this setting, we can get a flat PC from very small demand shocks or very large fixed adjustment costs. It is easy to show that κ is increasing in ξ. The numerator is increasing because $\xi(\xi - \xi_0)$ is, and since k^* is increasing, so is $F(k^*)$. If $k^* < \underline{k}$, then no firms will adjust prices and the PC will be perfectly flat. The denominator is decreasing by the same argument. Hence, smaller shocks lead to a flatter PC as fewer firms adjust.

Suppose firms start with marginal cost z_0, which then changes to z. This implies a cutoff $k^*(z)$, which is a different cost threshold for price-adjusting firms.

Lemma 5 (Inflation with Fixed Costs)**.** *With fixed costs of adjusting prices and a shock to marginal costs, $k^*(z) = \frac{\xi_0}{4}(z_0 - z)^2$, which implies*

$$\frac{\partial \bar{p}}{\partial z} = F(k^*(z))\frac{\xi}{2}.$$

Note that $k^*(\xi)$ is linear in ξ, while $k^*(z)$ is quadratic in z. Thus, if shocks are small (percentages), then $(z - z_0)^2$ is likely to be smaller than ξ, and thus for small shocks, $k^*(z) < k^*(\xi)$. This means that we are likely to get even less response to a comparably sized supply shock than to a demand shock. If few firms were adjusting in response to a demand shock, then few firms would be adjusting in response to a (comparable) cost shock. If z is near z_0, then the cutoff k^* is small and so few firms will adjust. Thus, it is completely plausible to have no response in this case as well. If $k^* < \underline{k}$, then no firms will adjust prices in response to a change in z.

Summary. To summarize, standard price-setting frictions cannot simultaneously generate a flat PC (κ close to zero) and a meaningful response of inflation to supply shocks (λ different from zero).

We consider quadratic adjustment costs (Rotemberg, 1982), contract frictions (Calvo or Taylor), and fixed adjustment costs. None of these standard settings produce a flat PC *as well as* significant responsiveness to inflationary shocks; κ and λ move together. A very flat PC requires very large exogenous frictions to change prices.

With standard price-setting frictions, variations in inflation would require implausibly large structural supply shocks, just as in the standard NK model. The intuition is straightforward: if it is exogenously costly to change prices, then it is costly to change prices in response to supply shocks as well as in response to demand shocks. In the case of Calvo or Taylor, if firms are stuck with a sticky price, then they cannot change it whether the disturbance is in demand or supply. These standard frictions cannot generate asymmetric price rigidity. We now present a microfoundation that can.

4 Strategically Sticky Prices

We now present a microfoundation in which price stickiness arises from the strategic behavior of firms reacting to an informational asymmetry between firms and consumers. We first consider the strategic behavior of firms in response to demand shocks and then consider their behavior in response to supply shocks.

In our model, the equilibrium consequences of an inflationary shock are quite different from those of a demand shock. First, firms have no incentive to use sticky prices in response to a shock to marginal cost, and as a result, the economy features a flexible-price equilibrium without an output gap (no distortions). Second, while prices are flexible with respect to the inflationary shock, prices may still be sticky with respect to changes in demand (for example, monetary policy). This last result is completely different from what would be predicted by standard models.

The intuition for these results is as follows. Cost-push shocks are perfectly transferred to prices because they do not lead to an increase in profits. Instead, because a demand increase leads to an increase in profits, it leads to a strategic interaction where consumers wonder if the price increase is justified. This is because, under information frictions, a subset of the consumers will suspect that the firm is cheating, and trying to increase unit profits even if the state of the world has not changed. As in L'Huillier (2020), this strategic interaction can push the firm to optimally post a price that, in equilibrium, does not adjust to the demand increase.[13]

[13] In fact, the optimal strategy of the firm is to post sticky prices when demand shifts (up or down) *and* when information frictions are severe.

4.1 Demand Shocks and Strategically Sticky Prices

The model we use is the same model that was set up in Section 3, with the addition of information frictions, and, for tractability, simplifying assumptions about aggregate risk.

Aggregate Risk. In the present, there is uncertainty about the aggregate state s of the economy, which we model as a shock to consumers' discount rate, denoted by ρ_s. For simplicity, we assume there are only two possible states, low L and high H, that occur with equal probability. As before, the aggregate state describes the level of demand *in the present*. We suppose the central bank responds to the aggregate state s by setting the nominal interest rate i_s setting $i_L \leq i_H$ in response to the endogenous levels of inflation and the output gap so that

$$\frac{1+\rho_L}{1+i_L} \equiv \xi_L < \xi_H \equiv \frac{1+\rho_H}{1+i_H}.$$

Hence, demand in the present will be *high* in state H. Section 5 considers equilibrium when the CB follows a Taylor rule for interest rates.

Islands: Consumer Types and Firms. Each island is populated by a continuum of consumers of total mass one and a single monopolistic firm. There are two types of consumers: Insiders (informed consumers) $\iota \in I$ and Outsiders (uninformed consumers) $o \in O$. Insiders are perfectly informed about the state; Outsiders are uninformed about the state but know the probability distribution and may draw inferences from the price set by the firm with which they trade.

The fraction $\alpha \in [0, 1)$ of Insiders on a particular island varies across islands. We assume the distribution of α is given by a cdf F whose support is not a singleton and has the property that $\lim_{\alpha \to 1} F(\alpha) = 1$. That is, the fraction of islands on which all consumers are Insiders is 0. Define α_0, α_1 to be the lower and upper limits of the support of F

$$\alpha_0 = \sup\{\alpha \in [0, 1] : F(\alpha) = 0\}$$
$$\alpha_1 = \inf\{\alpha \in [0, 1] : F(\alpha) = 1\}.$$

Hence, $[\alpha_0, \alpha_1]$ is the smallest closed interval that contains the support of F: α_0 is the fraction of Insiders on the least-informed island, and α_1 is the fraction of Insiders on the most-informed island. By assumption, the support of F is not a singleton, so $\alpha_0 < \alpha_1$.

Each island is inhabited by a single monopolistic firm with real marginal cost z. All firms know the true state and the fraction of Insiders on their island; firms can condition the price they set on the true state. The assumption that Insiders and firms know the true state is just a convenient abstraction of the idea that they are better informed than Outsiders.

Consumer Problem. Consumer j solves

$$\max_{c,C} \quad \mathbb{E}_j[(c - c^2/2) + \theta_s C]$$
$$\text{s.t. } pc + Q_s C = d_s + Q_s E,$$

where $\mathbb{E}_j[\ \cdot\ ]$ is consumer j's expectation operator at the present (conditioned on information available to that consumer), p is the nominal price the consumer faces in the short run for goods, and $Q_s \equiv \frac{1}{1+i_s}$ is the nominal price for bonds. Consumer j's optimal short-run demand will be

$$c^* = 1 - p\,\mathbb{E}_j\left[\frac{\theta_s}{Q_s}\right] \equiv 1 - p\,\mathbb{E}_j\left[\frac{1}{\xi_s}\right].$$

Hence, $\mathbb{E}_j\left[\frac{1}{\xi_s}\right]$ is the net demand shock determining whether demand in the present is strong or weak. As before, what matters for consumer demand is the *residual* demand net of the monetary policy response. Let ξ_0 denote the harmonic mean of ξ_L, ξ_H, that is,

$$\xi_0 \equiv \left[\frac{1}{2}\left(\xi_L^{-1} + \xi_H^{-1}\right)\right]^{-1}.$$

In the absence of any additional information

$$\mathbb{E}_j\left[\frac{1}{\xi}\right] = \frac{1}{\xi_0}.$$

Equilibrium Short-run Prices. Because each firm is a monopolist on its island, each firm sets the price for the consumption good. Absent any other considerations, it will be optimal for the firm to set the perfect information monopoly price in each state. It is easy to check that the perfect information monopoly (nominal) price and the resulting demand and (nominal) profit are

$$p_s = \frac{\xi_s(1+z)}{2}, \quad x_s(p_s) = \frac{1-z}{2}, \quad d_s(p_s) = \frac{\xi_s(1-z)^2}{4}$$

We abuse language and refer to the price schedule $\{p_s\}$ as the *flexible price.*

For each state s, the price p_s maximizes the firm's profit in state s, so the flexible price is optimal among all price schedules when information is perfect. However, if information is not perfect and consumers and firms behave strategically, this is not the only consideration; we must also ask whether the flexible price is consistent with equilibrium in the implicit game between the firm and the consumers. The question is whether adherence to the flexible price is optimal for the firm. For example, would the firm prefer to charge the price p_H even when the true state is L? It is possible that the firm will be tempted to charge p_H instead of p_L to extract more rents, especially if many consumers are Outsiders. In this context, the appropriate notion of equilibrium is Perfect Bayesian Equilibrium (PBE), so we should ask whether the flexible price is consistent with some PBE. This guarantees that the firm does not deviate from p_L in state L. Proposition 1 provides a sharp answer to this question.

Proposition 1 (PBE with Flexible Prices)**.** *The flexible price $\{p_s\}$ is consistent with some PBE if and only if*

$$\alpha \geq \bar{\alpha} \equiv \frac{1-z}{\frac{\xi_H}{\xi_L}(1+z) - 2z}. \tag{7}$$

When the fraction of Insiders is high enough, the flexible prices are in fact consistent with an equilibrium. As a note, $\bar{\alpha}$ is decreasing in z, and $\bar{\alpha} = \frac{\xi_L}{\xi_H}$ when marginal costs are zero.

The opposite end of the spectrum from the flexible price is a price schedule that is the same in both states of the world—a *sticky price*. For simplicity, we focus on a particular sticky price, denoted p_0, which is a natural choice for two reasons: it is the price that would be optimal if no consumers were informed ($\alpha = 0$), and it is the price that would be optimal in the absence of a shock (i.e., if $\xi_s = \xi_0$). Of course, we require that p_0 be consistent with some PBE as well; Proposition 2 provides a complete characterization.

Proposition 2 (PBE with Sticky Prices)**.** *The sticky price is $p_0 \equiv \xi_0(1+z)/2$. For z sufficiently small, p_0 is consistent with some PBE if $\alpha \leq \bar{\alpha}$.*

When the fraction of Insiders is low enough, the sticky price is in fact consistent with an equilibrium. Echoing what we said before: if too many consumers know the state, the sticky price is not a sustainable strategy. Finally, whenever *both* the sticky price and the flexible price schedule are consistent with PBE, the firm strictly prefers the flexible price schedule. We require that z not be so large because ξ affects both demand and nominal costs, and thus, a low z ensures that this latter effect does not dominate.

The Phillips Curve. The natural definition of the PC in this environment is as follows. Consider the two points $(\bar{x}_H, \bar{p}_H)$ and $(\bar{x}_L, \bar{p}_L)$. On each island, the firm chooses between the sticky price p_0 and the flexible price schedule $\{p_s\}$ defined above, subject to the requirement that whatever it chooses should be consistent with PBE and deliver the higher profit. On islands where $\alpha \geq \bar{\alpha}$, the firm will choose the flexible price schedule $\{p_s\}$ and demand will be $x_s(p_s) = \frac{1-z}{2}$. On islands where $\alpha < \bar{\alpha}$, the firm will choose the sticky price p_0 and demand will depend on the state. Keeping in mind that the Insiders know the state but the Outsiders do not, we see that on these islands demand will be

$$x_s(p_0) = \alpha \left[1 - p_0 \frac{1}{\xi_s}\right] + (1-\alpha)\left[1 - p_0 \frac{1}{\xi_0}\right].$$

(The first term is the demand of the Insiders, who know the true state; the second term is the demand of the Outsiders, who do not know the true state.) Having defined the local prices and demands in each state s, we define the *average* prices $\bar{p}_s$ and demands $\bar{x}_s$ in state s to be

$$\bar{p}_s = \int_0^{\bar{\alpha}} p_0 \,\mathrm{d}F(\alpha) + \int_{\bar{\alpha}}^1 p_s \,\mathrm{d}F(\alpha), \tag{8}$$

$$\bar{x}_s = \int_0^{\bar{\alpha}} \left[\alpha\left(1 - p_0 \frac{1}{\xi_s}\right) + (1-\alpha)\left(\frac{1-z}{2}\right)\right] \mathrm{d}F(\alpha) + \left(\frac{1-z}{2}\right)\left[1 - F(\bar{\alpha})\right]. \tag{9}$$

In this simple model, output is the average demand $\bar{x}_s$.

From equations (8) and (9), we can derive more convenient expressions for the average price difference and average demand difference:

$$\bar{p}_H - \bar{p}_L = \frac{1}{2}(\xi_H - \xi_L)(1+z)(1 - F(\bar{\alpha})), \tag{10}$$

$$\begin{aligned} \bar{x}_H - \bar{x}_L &= ([1 - p_0/\xi_H] - [1 - p_0/\xi_L]) \int_0^{\bar{\alpha}} \alpha \,\mathrm{d}F(\alpha) \\ &= \frac{\xi_0(1+z)}{2}\left(\frac{\xi_H - \xi_L}{\xi_H \xi_L}\right) \int_0^{\bar{\alpha}} \alpha \,\mathrm{d}F(\alpha). \end{aligned} \tag{11}$$

The price difference is the difference in the flexible prices for the measure of firms using flexible prices. The output difference is the difference in demand from Insiders on islands with sticky prices.

Therefore, the slope of the Phillips curve is

$$\kappa = \left(\frac{\xi_H \xi_L}{\xi_0}\right) \frac{1 - F(\bar{\alpha})}{\int_0^{\bar{\alpha}} \alpha \, \mathrm{d}F(\alpha)}. \tag{12}$$

We show that the PC flattens (i.e., the slope κ decreases) as the size of demand shocks decreases, and it becomes perfectly flat (i.e. $\kappa = 0$) in the limit as $\xi_H \to \xi_L$. Define $\delta \equiv \xi_H/\xi_L$. Note that $\delta \geq 1$ and decreases as $\xi_H \to \xi_L$. Define $\underline{\delta} \equiv \frac{1-z+2z\alpha_1}{\alpha_1(1+z)}$. We can state the following result.

Proposition 3 (Flattening of the Phillips Curve)**.** *The Phillips curve slope κ is increasing in δ, with a limit of $\kappa = 0$ when $\delta = 1$, and $\kappa = 0$ whenever $\delta < \underline{\delta}$. Thus, as $\xi_H \to \xi_L$ the Phillips curve flattens, and if ξ_H is sufficiently close to ξ_L, then the Phillips curve is completely flat.*

Proof. First, the coefficient $\left(\frac{\xi_H \xi_L}{\xi_0}\right)$ is strictly increasing in δ since ξ_0 is the harmonic mean; the coefficient decreases as $\delta \to 1$. Second, $\bar{\alpha}$ is decreasing in δ and equals 1 when $\delta = 1$; the fraction of islands posting sticky prices increases when δ is lower, converging to 1. Thus, the numerator decreases, the denominator increases, and δ decreases to 1. Therefore, κ is increasing in δ; as $\delta \to 1$ $\bar{\alpha} \to 1$ and all islands post sticky prices. If $\delta < \underline{\delta}$, then $\bar{\alpha} > \alpha_1$, which means that $F(\bar{\alpha}) = 1$, all islands post sticky prices, and $\kappa = 0$. □

To understand the mechanism underlying Proposition 3, note that, on all islands where sticky prices prevail, the PC (for those islands) is horizontal (has slope 0): prices are independent of the state but demands are not. Conversely, on all islands where flexible prices prevail, the PC (for those islands) is vertical (has slope $+\infty$): prices depend on the state but demands do not. As net demand shocks decrease, more firms choose sticky prices and fewer firms choose flexible prices. Fundamental demand shocks can still be large so long as the central bank policy offsets the shock, (i.e., raising rates in response to positive shocks, see L'Huillier, Phelan, and Zame, 2022).

There is also an "intensive margin" that is affected by the stability mandate. The slope of the PC as we have defined it is the ratio of the difference $\bar{p}_H - \bar{p}_L$ of (average) prices across states and the difference $\bar{x}_H - \bar{x}_L$ of (average) demands across states. As δ decreases, the flexible price prevails on fewer islands and the sticky price prevails on more islands—but the price differences across states and the demand differences across states *both* shrink. As δ tends toward 1, *both* the price differences and the demand differences tend toward 0, so the flattening of the PC—which is the ratio of these differences—depends not only on the fraction of firms and consumers that respond to the nominal shock but also on the *magnitudes* of those responses.

4.2 Supply Shocks and Strategically Flexible Prices

We now consider the consequences of an inflationary shock that directly affects firms' costs. Recall that the problem of standard models is having robust responsiveness of inflation to empirically plausible supply shocks when the Phillips curve is very flat.

Aggregate Risk and Prices. As in the previous section, we define a *supply shock* in this setting as changes in real marginal costs z. We can think of variations in z as capturing any variation in marginal costs, such as changes in the prices of intermediate goods or energy (like oil), changes in wages driven either by shocks to labor supply or shocks to wage-bargaining, or changes in productivity. In fact, whether the change in z represents an efficient or inefficient variation is not critical for our analysis.[14]

Suppose that the marginal cost can take two values, z_L or z_H, which reflect shocks to firm productivity. As we supposed with the demand shock, the firm and the Insiders know the value of z_s, but the Outsiders do not. Note that the Outsider demand depends only on the price p, which is observable, and the demand shifter ξ, which is known in this case. Thus, it is a straightforward result that the firm will always choose a flexible price, setting

$$p_s = \frac{\xi(1 + z_s)}{2},$$

and thus the economy features no output gap (output is at the flexible-price level, i.e., potential). In this economy, $\lambda = \partial p_s / \partial z$.

Proposition 4 (Flexible Prices with Inflationary Shocks)**.** *When the economy features a supply shock to the marginal cost z, all firms choose a flexible price $p_s = \frac{\xi(1+z_s)}{2}$, output is at potential, and $\lambda = \xi/2 > 0$.*

A few remarks are in order. First, in this model, in which prices are endogenously sticky for strategic reasons, demand shocks can lead to price stickiness while supply shocks will never lead to price stickiness. This means that the responsiveness of prices to inflationary shocks is completely different from the responsiveness to demand shocks.

Second, because the equilibrium is endogenously a flexible-price equilibrium, there is no welfare loss in response to the inflationary shock, and thus the optimal response of the central bank is to do nothing. In this model, there is no need for monetary policy to respond to the inflationary shock.

[14] The reason for this is that prices will be flexible when marginal costs move.

Nonetheless, central banks may have other reasons for wanting to respond to inflation in response to a shock to marginal costs. In the next section we show that if Outsiders do not know how the central bank will respond to an inflationary shock, then firms may adjust their prices in response to the inflationary shock but not in response to the central bank.

4.3 Supply and Demand Shocks

We now introduce both supply and demand shocks. With only supply shocks there is no well-defined Phillips curve (it is vertical) because output is always at potential. Introducing both sets of shocks, we can define a Phillips Curve slope with respect to demand and also the responsiveness of prices to supply shocks.

Aggregate Risk and Prices. Suppose the economy faces orthogonal supply and demand shocks: firms' marginal cost can take the values z_L or z_H, and residual demand can take the values ξ_H or ξ_L, for a total of four aggregate states. Let us clarify how the two shocks operate. Firms could possibly choose the following flexible prices, depending on the realizations of the states:

$$p_{H,H} = \frac{\xi_H(1+z_H)}{2}, \qquad p_{H,L} = \frac{\xi_L(1+z_H)}{2},$$
$$p_{L,H} = \frac{\xi_H(1+z_L)}{2}, \qquad p_{L,L} = \frac{\xi_L(1+z_L)}{2}.$$

Since firms have no strategic incentive to "hide" the realization of z_s, firms could choose the following sticky prices:

$$p_{0,H} = \frac{\xi_0(1+z_H)}{2}, \quad p_{0,L} = \frac{\xi_0(1+z_L)}{2},$$

where "stickiness" with respect to demand is evident in the ξ_0 term, while "flexibility" with respect to supply is evident in the z_s term.

Except by coincidence, these six prices are all distinct. This means that the firm can convey information about the shock z_s without conveying information about the demand shock ξ_s. When firms choose $p_{0,H}$ or $p_{0,L}$, we say that prices are flexible with respect to the supply shock but sticky with respect to demand. How sticky aggregate prices are with respect to demand depends on the fraction of firms choosing sticky prices. Note that $\bar{\alpha}$ depends on z. The aggregate price level given cost z and demand ξ is therefore

$$\bar{p}_{z,\xi} = \int_0^{\bar{\alpha}_z} \frac{\xi_0(1+z)}{2}\,\mathrm{d}F(\alpha) + \int_{\bar{\alpha}_z}^1 \frac{\xi(1+z)}{2}\,\mathrm{d}F(\alpha),$$
$$= \left(\frac{1+z}{2}\right)\left(\xi_0 + (\xi - \xi_0)(1 - F(\bar{\alpha}_z))\right), \tag{13}$$

where $\bar{\alpha}_z$ is parameterized by z. As before, firms choose flexible prices if $\alpha > \bar{\alpha}_z$. Thus, the aggregate price $\bar{p}_{z,\xi}$ is shifted directly by the cost z but also depends on demand through the fraction of firms posting flexible prices, determined by $\bar{\alpha}_z$.

Because the Phillips curve captures the relationship between prices and demand pressure, the appropriate definition is to consider two curves parameterized by the supply shock z relating how variations in demand ξ affect prices. When the supply shock is high, we have one curve with slope κ_H connecting $(\bar{x}_{H,H}, \bar{p}_{H,H})$ and $(\bar{x}_{H,H}, \bar{p}_{H,H})$. When the supply shock is low, we have a curve with slope κ_L connecting $(\bar{x}_{L,H}, \bar{p}_{L,H})$ and $(\bar{x}_{L,H}, \bar{p}_{L,H})$. Defining $\bar{\alpha}_H \equiv \frac{1-z_H}{\frac{\xi_H}{\xi_L}(1+z_H)-2z_H}$ and $\bar{\alpha}_L \equiv \frac{1-z_L}{\frac{\xi_H}{\xi_L}(1+z_L)-2z_L}$, the slopes of the PCs are

$$\kappa_H = \left(\frac{\xi_H \xi_L}{\xi_0}\right)\frac{1-F(\bar{\alpha}_H)}{\int_0^{\bar{\alpha}} \alpha\ \mathrm{d}F(\alpha_H)}, \quad \kappa_L = \left(\frac{\xi_H \xi_L}{\xi_0}\right)\frac{1-F(\bar{\alpha}_L)}{\int_0^{\bar{\alpha}} \alpha\ \mathrm{d}F(\alpha_L)},$$

where the subscript denotes the value of the supply shock. The levels of prices are shifted by the cost z, and the slopes in the two states differ because $\bar{\alpha}$ depends on z. Recall that $\bar{\alpha}$ is decreasing in z and therefore $\bar{\alpha}_H < \bar{\alpha}_L$; more firms choose sticky prices when the supply shock is low. Define $\underline{\delta}_H \equiv \frac{1-z_H+2z_H\alpha_1}{\alpha_1(1+z_H)}$. The following is an immediate consequence.

Proposition 5 (Inflation and Supply Shocks)**.** *When the economy features both supply and demand shocks, aggregate prices are flexible with respect to the supply shock but can be sticky with respect to the demand shock. If $\delta < \underline{\delta}_H$, then $\kappa_H = \kappa_L = 0$; the Phillips curve is perfectly flat with respect to demand shocks, but supply shocks create meaningful inflation.*

This last result is crucial. Our model predicts that strategic price stickiness can lead to a completely flat Phillips curve with $\kappa = 0$ and also that supply shocks can meaningfully feed through to inflation. Notice that for the sticky price $p_{0,s}$, we have $\partial p_{0,s}/\partial z = \xi_0/2 > 0$. For all $\alpha > \alpha_0$, some firms will choose prices that are sticky with respect to demand, but all firms will choose prices that are flexible with respect to supply. Thus, the responsiveness of prices to supply shocks will always exceed the responsiveness to demand shocks. Furthermore, if residual demand shocks are not very volatile, then the PC will be completely flat, with all firms choosing prices that are sticky with respect to demand but flexible with respect to supply.

Put differently, we can linearize (13) in terms of deviations from the mean shocks as

$$\hat{p}_t = \widehat{z}_t\,(\xi_0/2) + \widehat{\xi}_t(1 - F(\bar{\alpha})), \tag{14}$$

which is the analog to the standard NK PC written in terms of the structural shocks directly. When the PC is flat, $F(\bar{\alpha}) = 1$, and thus $\hat{p}_t$ is not at all responsive to demand shocks $\widehat{\xi}_t$ but is very responsive to supply shocks $\widehat{z}_t$, which are structural. In sum, our model can endogenously produce a perfectly flat PC with inflation that is responsive to structural supply shocks.

Monetary policy as the source of demand shocks. Now suppose that the central bank may respond to a supply shock with accommodative or contractionary policy, resulting in the demand shifter being either ξ_H or ξ_L, where the demand level now reflects the policy choice of the central bank alone. Accommodative policy (low rates) yields ξ_H, while contractionary policy (high rates) yields ξ_L. Monetary policy is relatively inactive when ξ_L and ξ_H are close.

In either case, we suppose that the CB's action is independent of the cost shock. Let us explain the assumption that the CB's action is independent of the state. What is important is that Outsiders do not ex-ante know the stance of monetary policy, even if they know the marginal cost z_s (which they will in equilibrium). Modeling monetary policy as an independent decision preserves that. All that is required is that Outsiders face residual uncertainty about monetary policy conditional on the inflationary shock.

There are thus two effects of monetary policy on the price level. First, the flexible price is directly proportional to ξ_s, so contractionary policy (implementing ξ_L) lowers the flexible price picked by firms. Second, monetary policy determines which islands use sticky prices.

All else being equal, more firms choose sticky prices when the CB response is muted (when ξ_L and ξ_H are close). If the CB response is muted, then a contractionary policy to bring down inflation would not have a very large effect because only islands with $\alpha \geq \bar{\alpha}$ would lower prices in response. However, the larger the CB response (the bigger the difference between ξ_L and ξ_H), the greater the extensive margin: more islands would choose the flexible price.

Thus, when responding to an inflationary shock alone, the effect of monetary policy on the price level is stronger than linear: the intensive margin is linear in the policy ξ_s, and more aggressive policy increases the extensive margin, inducing more firms to choose flexible prices. The following result is an immediate corollary of of Proposition 5.

Corollary 1 (Inflationary Shocks and Monetary Policy)**.** *When the economy features a shock to the marginal cost z and Outsiders do not know how monetary policy might respond, aggregate prices are flexible with respect to the cost shock but sticky with respect to monetary policy.*

With more aggressive monetary policy, more firms choose flexible prices, thus strengthening the intensive and extensive margins decreasing the price gap. However, more aggressive monetary policy could *increase* the output gap. If many firms choose sticky prices, then monetary policy, which changes the demand of Insiders only, creates distortions and thus an output gap. Hence, the central bank may face the standard tradeoff in responding to an inflationary shock: closing the price gap may require increasing the output gap.

5 Quantitative Implications and Monetary Policy

We now provide a calibrated model to determine the quantitative significance of our theoretical results. We have shown that it is theoretically possible to have a flat Phillips curve in this setting and for supply shocks to cause inflation. We now show that our proposed mechanism produces an empirically realistic Phillips curve given a reasonable calibration. We then use our calibrated model to consider the aggregate consequences of supply shocks when monetary policy responds to supply shocks.

5.1 Setup and Calibration

For this section, we modify the setup slightly in order to let the data discipline the degree of heterogeneity. First, instead of having a distribution of Insiders across islands, we let firms have a distribution of marginal costs. In this way, we can let empirical estimates discipline the distribution of productivity. Second, we explicitly model the behavior of the central bank using a Taylor rule that determines interest rates in equilibrium. Finally, we calibrate the fraction of Insiders to match the estimated slope of the Phillips Curve, given particular Taylor coefficients disciplined by the data.

Firms, Aggregation, and the Phillips Curve. Let the setup be modified as follows. The fraction of Insiders α is assumed to be constant across islands, but now firms differ in their marginal costs $z \leq 1$. Let marginal costs be distributed according to $z \sim G$, where G is a CDF with the usual properties. Because α is constant across islands, the cutoff for sticky or flexible prices now depends on the marginal cost, denoted by $\bar{z}$.

Proposition 6 (PBE with Flexible Prices and Marginal Costs)**.** *The flexible price* $\{p_s\}$ *is consistent with some PBE if and only if*

$$z \geq \bar{z} \equiv \frac{1-\alpha\xi_H/\xi_L}{\alpha(\xi_H/\xi_L-2)+1}.$$

The *average* prices $\bar{p}_s$ and demands $\bar{x}_s$ in state s can be written

$$\bar{p}_s = \int_0^{\bar{z}} \xi_0 \frac{1+z}{2}\,\mathrm{d}G(z) + \int_{\bar{z}}^1 \xi_s \frac{1+z}{2}\,\mathrm{d}G(z), \tag{15}$$

$$\bar{x}_s = \int_0^{\bar{z}} \left(\alpha\left[1-\frac{1+z}{2}\frac{\xi_0}{\xi_s}\right] + (1-\alpha)\left[\frac{1-z}{2}\right]\right)\mathrm{d}G(z) + \int_{\bar{z}}^1 \frac{1-z}{2}\mathrm{d}G(z). \tag{16}$$

Therefore, the slope of the Phillips curve is

$$\kappa = \left(\frac{\xi_H\xi_L}{\alpha\xi_0}\right)\frac{\int_{\bar{z}}^1 (1+z)\,\mathrm{d}G(z)}{\int_0^{\bar{z}}(1+z)\;\mathrm{d}G(z)}. \tag{17}$$

Compared to the baseline model, we aggregate using G instead of F; the model behaves similarly.[15]

Monetary Policy. We suppose the CB responds to the aggregate state s by setting the nominal interest rate i_s according to a standard Taylor rule, which we write

$$i_s^{Taylor} = i_0 + \phi_\pi(\bar{p}_s - \bar{p}_0) + \phi_x\left(\bar{x}_s - \bar{x}_0\right), \tag{18}$$

where $\bar{p}_s$ is the aggregate price level (inflation), $\bar{x}_s$ is aggregate output, i_0 is a base interest rate, ϕ_π and ϕ_x are the Taylor coefficients, and $\bar{p}_0$ and $\bar{x}_0$ are base measures of prices and output. Therefore, $\bar{p}_s - \bar{p}_0$ is the deviation of inflation from baseline and $\bar{x}_s - \bar{x}_0$ is the output gap. In equilibrium this means the CB will endogenously set $i_L \leq i_H$ in response to the endogenous levels of inflation and the output gap.

In practice, monetary policy rules typically respond to output gaps and to inflation, rather than to a directly observable shock. Therefore, because policy makers do not observe shocks directly,

[15] As an aside, note that in this setting a supply shock that increases the distribution of z would increase the fraction of flexible-price firms, which would all else being equal steepen the Phillips curve. Because this is not the main point of our paper, we do not emphasize this result, but leave it to later work to further investigate this prediction.

the policy rule responds indirectly through observable macro variables. The Taylor coefficients ϕ_π and ϕ_x will endogenously determine how much the CB changes i_s in response to the shock.

Solving for equilibrium requires solving a fixed-point problem. Note that the interest rate i_s determines ξ_s, which affects average prices and output—which are themselves the inputs in the Taylor Rule. All else being equal, when the Taylor coefficients are large, the CB will respond aggressively to variations in output and prices (offsetting the demand shock), which will endogenously lead to smaller fluctuations in output and demand. In this way, the slope of the Phillips curve is determined by the Taylor rule (see L'Huillier, Phelan, and Zame, 2022).

Calibration. We let the data discipline the degree of heterogeneity in the model, thus calibrating the slope of the PC in equilibrium.

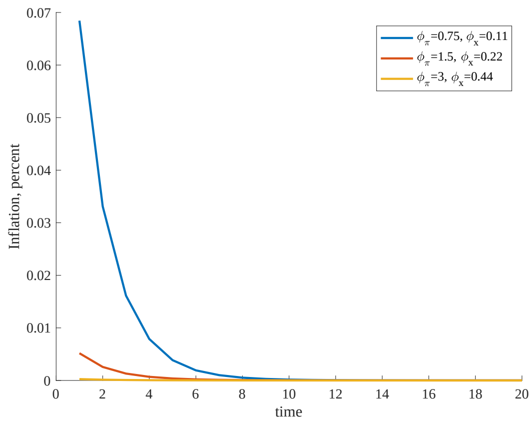
We target average markups to be 12.5%, as is standard in the literature. With an average price of $p = \frac{\xi_0(1+z)}{2}$, we set $p/z = 1.125$ and $\xi_0 = 1$, implying an average marginal cost of 0.8. We calibrate the distribution of productivities (inverse of marginal costs) as log-normal with standard deviation of 5%. Bloom et al. (2018) find that the unconditional standard deviation of micro-productivity shocks is 5.1%. The mean is set so that the average marginal cost equals 0.8.

The household time preference (natural rate) is set to $r_0 = 4\%$, and the discount factor shock (demand shock) is 1%. Therefore, $\theta_L = \frac{1.01}{1+r_0}$ and $\theta_H = \frac{0.99}{1+r_0}$.

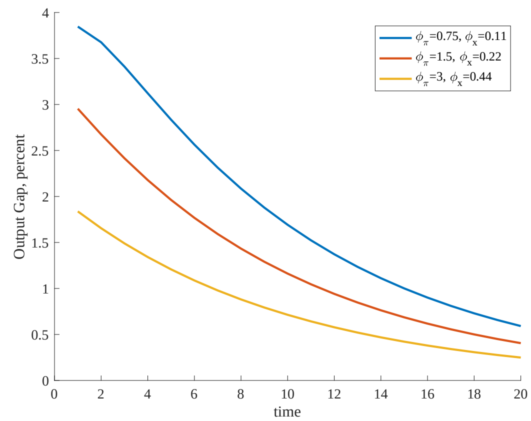
We choose α to target the slope of the PC in equilibrium for the given Taylor coefficients. We use estimates from Del Negro, Lenza, Primiceri, and Tambalotti (2020) who use 1990 as the break in the sample. Their estimates for the relevant Taylor coefficients post-1990 are $\phi_\pi = 1.5$ and $\phi_x = 0.22$. The posterior mean, median, and mode for κ post-1990 are 0.00151, 0.00140, and 0.00196. These estimates are similar to what is found in Hazell, Herreño, Nakamura, and Steinsson (2022). With $\alpha = 0.88$, equilibrium features an equilibrium Phillips Curve slope of $\kappa = 0.0017$. In other words, the calibrated model can match the slope of the Phillips Curve given the estimated the Taylor rule parameters.

5.2 Shocks and Monetary Policy Responses

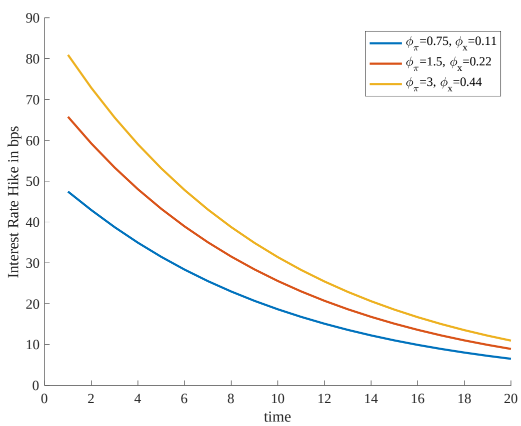
A calibrated version of our model can produce a very flat Phillips curve consistent with empirical estimates. Equilibrium features endogenous monetary policy responses and firms with heterogeneous productivities disciplined by empirical estimates. We now consider the aggregate dynamics of a 1% structural demand shock and a 1% structural supply shock (an increase in marginal costs of 1%), each decaying at a rate of 0.9. We shut down learning dynamics so that $\alpha = 0.88$ is constant across time. In this model, demand shocks produce virtually no change in average prices. In contrast, a 1% aggregate productivity shock would change marginal costs by 1%, which changes aggregate prices by 0.45% on impact in our model. Thus, empirically plausible supply shocks can produce meaningful inflation even while demand shocks produce a very flat Phillips curve.



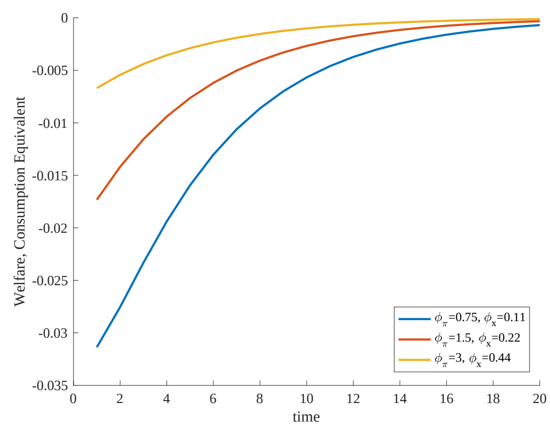
(a) Inflation



(b) Output Gap



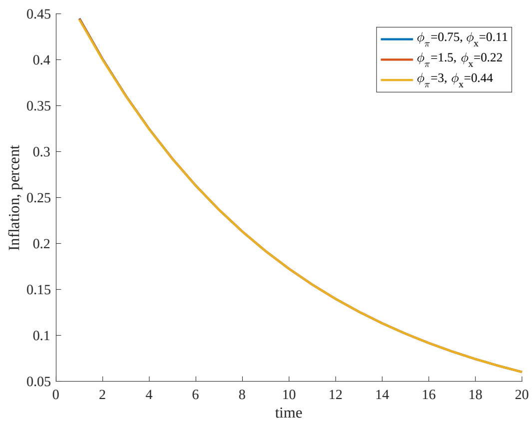
(c) Interest Rate



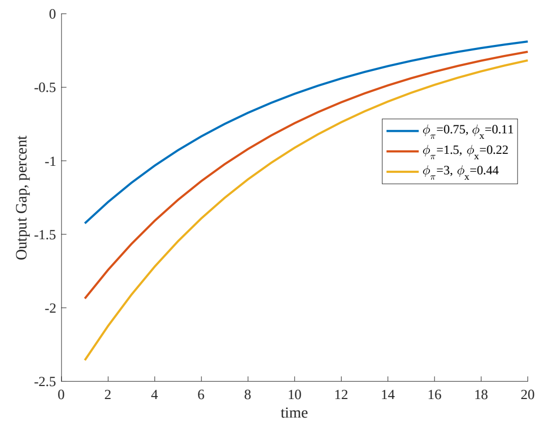
(d) Welfare (CE)

Figure 1: Aggregate consequences of a demand shock. Inflation, output, interest rates, and welfare, varying the aggressiveness of the monetary policy rule. Red: Baseline Taylor rule; Blue: Mild MP with Taylor coefficients times 1/2; Yellow: Aggressive MP with Taylor coefficients times 2. Source: Authors' analysis.

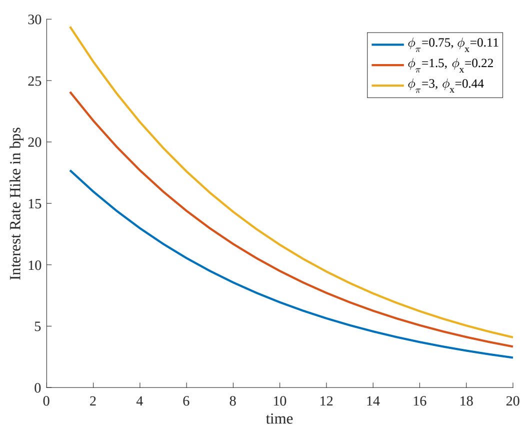
We first consider a decaying demand shock. Figure 1 plots inflation, the output gap, interest rates, and welfare in terms of consumption-equivalents relative to the flexible-price allocation, given a 1% demand shock. We also consider variation in monetary policy, by considering more and less aggressive Taylor rules. A Mild MP response follows a Taylor rule with coefficients multiplied by 1/2 and an Aggressive MP response follows a Taylor rule with coefficients multiplied by 2. The demand shock generates a positive output gap and negligible inflation; the least aggressive monetary policy response leads to an increase in inflation of 0.07% on impact. As is standard, more aggressive central bank response decreases the welfare losses from the output gap.



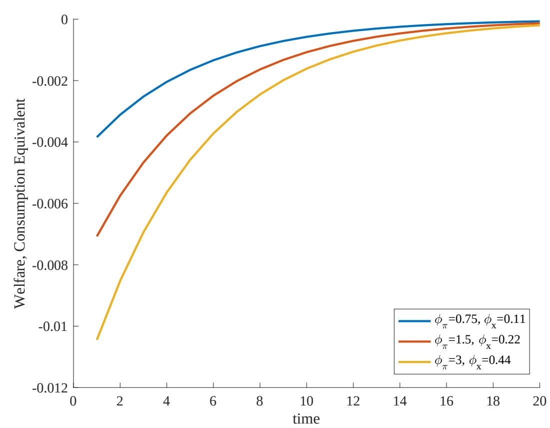
(a) Inflation



(b) Output Gap



(c) Interest Rate

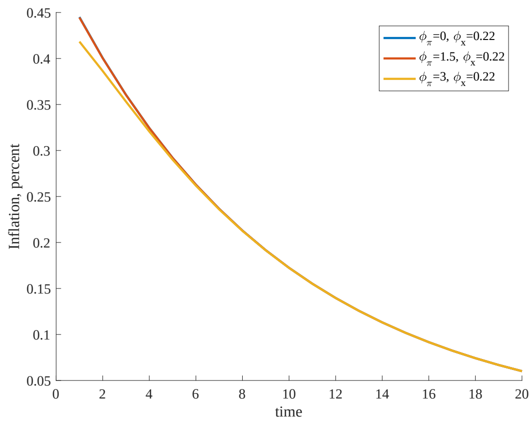


(d) Welfare (CE)

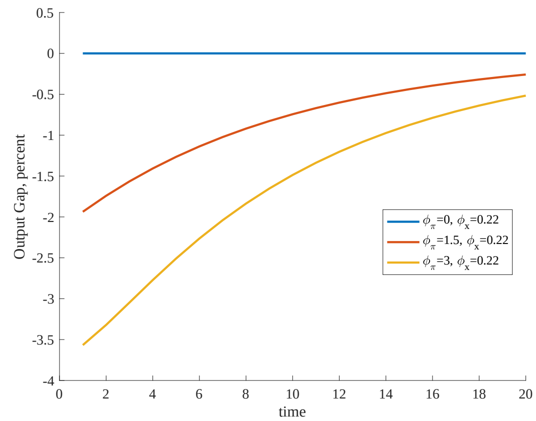
Figure 2: Aggregate consequences of a supply shock. Inflation, output, interest rates, and welfare, varying the aggressiveness of the monetary policy rule. Red: baseline Taylor rule; Blue: Mild MP with Taylor coefficients times 1/2; Yellow: Aggressive MP with Taylor coefficients times 2. Source: Authors' analysis.

Figure 2 instead considers the aggregate consequences of a structural supply shock. The theoretical result of our paper is that a supply shock leads to a flexible change in prices with respect to the cost shock, but prices can remain sticky with respect to changes in demand, which in this case would be variations in monetary policy. Accordingly, a 1% structural supply shock leads to a 0.45% increase in inflation on impacts (orders of magnitude larger than the effects from a demand shock) and meaningful negative output gaps in response to the increase in interest rates. The negative output gap leads to welfare losses. In this case, more aggressive monetary policy is not welfare-improving.[16]

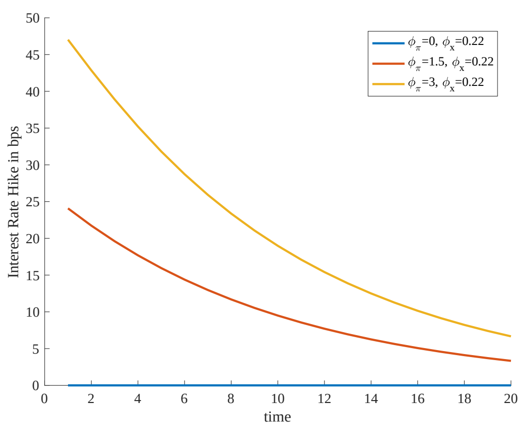
Crucially, the inflation outcomes in all three scenarios are virtually identical: the central bank response has virtually no effect on inflation. Importantly, an aggressive monetary policy is not enough to bring down inflation.



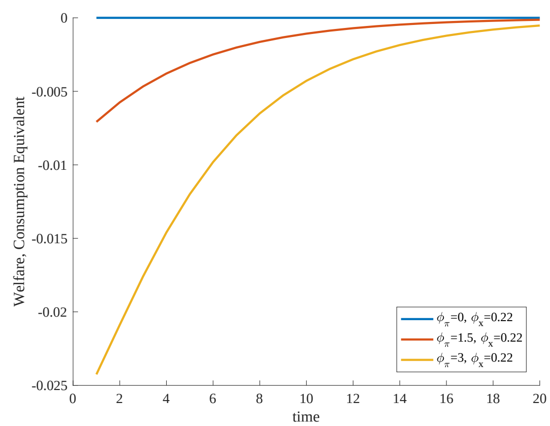
(a) Inflation



(b) Output Gap



(c) Interest Rate



(d) Welfare (CE)

Figure 3: Monetary policy responses to a supply shock. Inflation, output, interest rates, and welfare, varying the hawkishness of the monetary policy rule. Red: Baseline Taylor rule; Blue: Dovish, $\phi_\pi = 0$; Yellow: Hawkish, $\phi_\pi = 3$. Source: Authors' analysis.

Figure 3 further investigates this issue. Rather than varying both Taylor coefficients, we now

[16]Note that in this model, the supply shock z leads to positive inflation and a negative output gap—a cost-push shock, even though the supply shock need not be a mark-up shock, as in the NK model, and could instead reflect a productivity shock. In the NK model, a productivity shock leads inflation and the output gap to move in the same direction. The reason for this, again, is the demand-supply asymmetry of price stickiness in our model, combined with the reaction of monetary policy to inflation.

vary the aggressiveness of the central bank in responding to inflation by varying ϕ_π alone while fixing ϕ_x at the baseline. In the hawkish case, the central bank has twice the response to inflation without changing its weight on the output gap. In the dovish case, we set $\phi_\pi = 0$ and in equilibrium there is no monetary policy response to a supply shock.

The results in this case are even starker. When the central bank does not respond at all to the supply shock, there is no output gap (flexible-price equilibrium), and therefore there are no welfare losses. Even a very hawkish response that puts twice the baseline weight on inflation leads to almost no change in inflation. An extremely hawkish response, tripling of the Taylor coefficient to $\phi_\pi = 4.5$, would raise rates in equilibrium by 60 basis points in response to a 1% supply shock and would still yield an increase in inflation of 0.32% on impact while creating a negative output gap of almost 4%.

Our results suggest that a central bank attempting to bring down inflation in response to a supply shock faces a daunting task. Because the Phillips curve is very flat, a very aggressive response in interest rates is likely to have a large negative effect on output without a significant effect on inflation.

In our simple benchmark model, our theoretical results suggest that the increase in prices in response to a supply shock would be efficient; the central bank should not respond by raising rates. In reality, there are likely to be other frictions and rigidities in the economy so that inflation may be costly. We have left out, for example, the possibility of embedded inflation expectations responding in adverse ways. Our analysis nonetheless highlights the policy challenges in responding to inflation in the event that supply shocks lead to efficient inflation. Nonetheless, to the extent that prices are less rigid in response to supply shocks, the welfare losses due to nominal rigidities would necessarily be lower. The next section provides empirical evidence that, indeed, aggregate prices are more flexible with response to cost shocks, suggesting that the welfare considerations of supply shocks are lower than those of demand shocks.

6 Empirical Evidence

In this section, we provide U.S. time-series evidence supporting the view that cost-push shocks lead to stronger short-run inflation responses than demand shocks. Specifically, we identify both of these shocks using a state-of-the-art procedure. For a similar effect on U.S. industrial production (IP), we show that whereas demand shocks lead to a relatively small inflation response in the short run, cost-push shocks deliver a response that is about 2.5 times larger over two years and more than 5 times larger over one year.

Our empirical approach is fairly off-the-shelf. The simple time-series exercise we present essentially collects and combines findings from recent studies. Both types of shocks are identified using external instruments. In order to identify the effect of demand shocks on inflation and output, we consider well-identified monetary shocks based on the instrument proposed by Gertler and Karadi (2015). We follow their procedure closely by running a VAR on log IP, the log consumer price index (CPI), the one-year government bond rate (as the policy indicator), and a credit spread, and by using the three-month-ahead funds rate future surprise to identify monetary policy shocks.

In order to identify the effect of cost-push shocks on inflation and output, we consider well-identified oil news shocks based on Känzig (2021). Here, we also follow his procedure closely by running a VAR on the real price of oil, world oil production, world oil inventories, world IP, U.S. industrial production, and the log CPI, and by using his series of high-frequency surprises around OPEC announcements to identify oil shocks.

For both exercises, the data are monthly and the sample spans 1983:4 through 2017:12. Both VARs have 12 lags. Having identified the shocks, we compute the impulse response functions of IP and inflation. As expected, following a monetary shock, IP and inflation co-move, but they do not following a cost-push shock. For both types of shocks, we consider a shock that raises inflation (i.e., an expansionary monetary shock and a contractionary cost-push shock). We set the size of an oil news shock to one standard deviation and compute the responses of IP and CPI. We then scale the size of a monetary shock as follows: we compute the IP response after the cost-push shock at a horizon of 24 months, and then we re-size the monetary shock to deliver the same IP response at the same horizon (in absolute value).

Figure 4 presents the results. It plots the impulse response functions (IRFs) at the point estimate, and the corresponding 68% error bands. Looking at the monetary shock (the dashed, red line), we see that IP rises gradually and reaches a 0.60% increase in 24 months. The CPI raises by roughly 0.10% on impact of the shock, and then stays roughly constant over the horizon considered. For the cost-push shock (the solid, blue line), we estimate a gradual decline in IP, with a fairly rapid rise in the CPI that peaks at 0.30% at 12 months. The inflation response drops slightly thereafter, settling at 0.25% after 24 months.

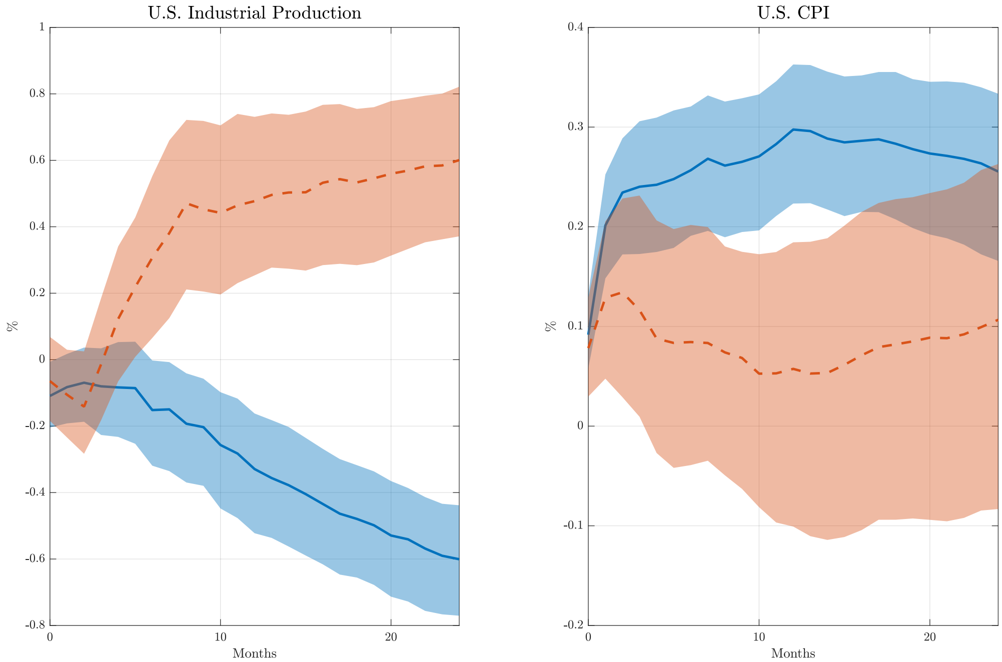


Figure 4: Estimated responses of U.S. Industrial Production and the Consumer Price Index to a monetary shock (dashed, red line), and to a cost-push shock (solid, blue line). The cost-push shock is re-sized to deliver the an IP response of the same size at 24 months. Source: Gertler and Karadi (2015), Känzig (2021), and authors' analysis.

Overall, for a similar effect on IP, we note that the response of inflation in the case of the cost-push is roughly 2.5 times larger than in the case of the monetary shock over 24 months and more than 5 times larger over 12 months. Moreover, the response in the case of the cost-push shock is statistically different from zero at all horizons, whereas the response to the monetary shock is actually not different from zero for the majority of the response.

7 Conclusion

Standard models cannot account for a very flat Phillips curve and also meaningful inflation caused by empirically plausible supply disturbances. We argue that this is because standard models treat price rigidities with respect to demand and supply symmetrically, which suggests that price setting frictions with asymmetric frictions have a better chance of matching the empirical facts. We provide a microfoundation of price stickiness based on strategic behavior of informed firms that can simultaneously produce both a very flat Phillips curve and also inflation in response to supply shocks. When net demand shocks are not so volatile, firms strategically choose sticky prices that do not fluctuate in response to demand. In contrast, firms have no strategic incentive to choose prices that do not fluctuate in response to marginal costs, and therefore prices are flexible with respect to changes in supply.

Our model is able to reconcile the important empirical facts that the Phillips curve is very flat and yet the economy features meaningful inflation. Our model does not need to resort to implausibly large structural shocks to firms' pricing decisions. Prices can be completely flexible with respect to supply shocks and yet remain rigid with respect to demand shocks, including changes in monetary policy. A calibrated version of our model shows that a 1% structural cost shock could increase inflation on impact by 0.45% (orders of magnitude larger than a standard New Keynesian model) and suggests that central banks might not want to respond to supply shocks.

More research is required on the consequences of asymmetric price rigidity. Our baseline model does not incorporate any other frictions aside from nominal rigidities at the level of firms' pricing decisions. Future work ought to consider the role of wage rigidities, which are unlikely to share the same microfoundation as we have proposed at the level of product prices.

References

Acharya, S. (2017): "Costly Information, Planning Complementarities, and the Phillips Curve," *Journal of Money, Credit and Banking*, 49, 823–850.

Ascari, G. (2004): "Staggered prices and trend inflation: some nuisances," *Review of Economic Dynamics*, 7, 642–667.

Ascari, G., P. Bonomolo, and Q. Haque (2022): "The Long-Run Phillips Curve is... a Curve," Tech. rep.

Ascari, G. and L. Fosso (2021): "The Inflation Rate Disconnect Puzzle: On the International Component of Trend Inflation and the Flattening of the Phillips Curve," Tech. rep.

Ascari, G. and T. Haber (2022): "Non-linearities, state-dependent prices and the transmission mechanism of monetary policy," *The Economic Journal*, 132, 37–57.

Ball, L., D. Leigh, and P. Mishra (2022): "Understanding US Inflation during the COVID Era," Tech. Rep. No. w30613, NBER Working Paper.

Ball, L., N. G. Mankiw, and R. Reis (2005): "Monetary policy for inattentive economies," *Journal of Monetary Economics*, 52, 703–725.

Benigno, P. and L. A. Ricci (2011): "The Inflation-Output Trade-Off with Downward Wage Rigidities," *American Economic Review*, 101, 1436–66.

Bernstein, J. and R. Kamdar (2022): "Rationally Inattentive Monetary Policy," *Review of Economic Dynamics*.

Blinder, A. (1991): "Why Are Prices Sticky? Preliminary Results from an Interview Study," *American Economic Review*, 89-96.

Blinder, A., E. Canetti, D. Lebow, and J. Rudd (1998): *Asking About Prices: A New Approach to Understanding Price Stickiness*, New York, Russell Sage Foundation.

Bloom, N., M. Floetotto, N. Jaimovich, I. Saporta-Eksten, and S. J. Terry (2018): "Really uncertain business cycles," *Econometrica*, 86, 1031–1065.

Bunn, P., L. S. Anayi, N. Bloom, P. Mizen, G. Thwaites, and I. Yotzov (2022): "Firming up price inflation," Tech. rep., National Bureau of Economic Research.

Caballero, R. J. and E. M. Engel (2007): "Price stickiness in Ss models: New interpretations of old results," *Journal of monetary economics*, 54, 100–121.

Cavallo, A. (2018): "More Amazon effects: online competition and pricing behaviors," *NBER WP 25138*.

Chari, V. V., P. J. Kehoe, and E. R. McGrattan (2009): "New Keynesian Models: Not Yet Useful for Policy Analysis," *American Economic Journal: Macroeconomics*, 1, 242–66.

Coibion, O., Y. Gorodnichenko, and R. Kamdar (2018): "The formation of expectations, inflation, and the phillips curve," *Journal of Economic Literature*, 56, 1447–91.

Del Negro, M., M. Lenza, G. E. Primiceri, and A. Tambalotti (2020): "What's up with the Phillips curve?" *BPEA Conference Drafts*.

Di Giovanni, J., Ș. Kalemli-Özcan, A. Silva, and M. A. Yildirim (2022): "Global Supply Chain Pressures, International Trade, and Inflation," Tech. Rep. No. 30240, National Bureau of Economic Research.

Forbes, K. J. (2019): "Inflation dynamics: dead, dormant, or determined abroad?" *Brookings Papers on Economic Activity*.

Gaballo, G. and L. Paciello (2021): "Spending Allocation under Nominal Uncertainty: A Model of Effective Price Rigidity," Tech. rep., CEPR Discussion Paper No. DP16101.

Galí, J. (2015): *Monetary policy, inflation and the business cycle: An introduction to the New Keynesian framework*, Princeton University Press.

Gertler, M. and P. Karadi (2015): "Monetary Policy Surprises, Credit Costs, and Economic Activity," *American Economic Journal: Macroeconomics*, 7, 44–76.

Gilchrist, S. and E. Zakrajsek (2012): "Credit Spreads and Business Cycle Fluctuations," *American Economic Review*, 102, 1692–1720.

Greenwald, B. and J. E. Stiglitz (1989): "Toward a theory of rigidities," *The American Economic Review*, 79, 364–369.

Gutiérrez-Daza, Á. (2022): "Business Cycles when Consumers Learn by Shopping," .

Hall, R. L. and C. J. Hitch (1939): "Price theory and business behaviour," *Oxford economic papers*, 12–45.

Harding, M., J. Lindé, and M. Trabandt (2022a): "Resolving the missing deflation puzzle," *Journal of Monetary Economics*, 126, 15–34.

——— (2022b): "Understanding Post-Covid Inflation Dynamics," Tech. rep., Working Paper.

Hazell, J., J. Herreño, E. Nakamura, and J. Steinsson (2022): "The Slope of the Phillips Curve: Evidence from U.S. States," *The Quarterly Journal of Economics*, 137, 1299–1344.

Jorgensen, P. L. and K. J. Lansing (2019): "Anchored inflation expectations and the flatter Phillips curve," *Federal Reserve Bank of San Francisco WP 2019-27*.

Kahneman, D., J. L. Knetsch, and R. H. Thaler (1986): "Fairness as a constraint on profit seeking: Entitlements in the market," *American Economic Review*, 76, 728–741.

Känzig, D. R. (2021): "The Macroeconomic Effects of Oil Supply News: Evidence from OPEC Announcements," *American Economic Review*, 111, 1092–1125.

Karadi, P., R. Schoenle, and J. Wursten (2022): "Price Selection in the Microdata," Tech. rep., CEPR Press Discussion Paper No. 15383.

Kiley, M. T. (2000): "Endogenous price stickiness and business cycle persistence," *Journal of Money, Credit and Banking*, 28–53.

Kocherlakota, N. R. (2021): "The Unavoidability of Low Inflation–Low Output Traps," *The Review of Economic Studies*.

Lagos, R. and R. Wright (2005): "A Unified Framework for Monetary Theory and Policy Analysis," *Journal of Political Economy*, 113, 463–484.

Levin, A. and T. Yun (2007): "Reconsidering the natural rate hypothesis in a New Keynesian framework," *Journal of Monetary Economics*, 54, 1344–1365.

L'Huillier, J.-P. (2020): "Consumer Imperfect Information and Endogenous Price Rigidities," *American Economic Journal: Macroeconomics (forthcoming)*.

L'Huillier, J.-P., G. Phelan, and W. R. Zame (2022): "Price Stability Can Flatten the Phillips Curve," Working paper, Brandeis University.

L'Huillier, J.-P. and R. Schoenle (2019): "Raising the Inflation Target: How Much Extra Room Does It Really Give?" Mimeo.

L'Huillier, J.-P. and W. R. Zame (2022): "Optimally sticky prices: Foundations," *Journal of Economic Dynamics and Control*, 104397.

Lucas, R. E. and N. L. Stokey (1987): "Money and interest in a cash-in-advance economy," *Econometrica*, 55, 491–513.

Mankiw, N. G. and R. Reis (2002): "Sticky information versus sticky prices: A proposal to replace the New Keynesian Phillips curve," *The Quarterly Journal of Economics*, 117, 1295–1328.

McLeay, M. and S. Tenreyro (2020): "Optimal Inflation and the Identification of the Phillips Curve," *NBER Macroeconomics Annual*, 34, 199–255.

Nakamura, E. and J. Steinsson (2011): "Price Setting in a Forward-Looking Customer Markets," *Journal of Monetary Economics*, 58, 220–233.

Okun, A. M. (1981): *Prices and Quantities: A Macroeconomic Analysis*, Washington, DC: Brookings Institution.

Petrosky-Nadeau, N. and B. Bundick (2021): "From Deviations to Shortfalls: The Effects of the FOMC's New Employment Objective," *Federal Reserve Bank of Kansas City Working Paper*.

Rotemberg, J. J. (1982): "Monopolistic price adjustment and aggregate output," *Review of Economic Studies*, 49, 517–531.

——— (2005): "Customer anger at price increases, changes in the frequency of price changes and monetary policy," *Journal of Monetary Economics*, 52, 829–852.

Shapiro, A. H. (2022): "How Much do Supply and Demand Drive Inflation?" *FRBSF Economic Letter*, 15.

Werning, I. (2022): "Expectations and the Rate of Inflation," Tech. Rep. No. 30260, National Bureau of Economic Research.

Appendices

A Proofs

A.1 Proofs for Section 3.2, Standard Price-setting Frictions

Proof of Lemma 2. Profits are

$$p - z\xi - p^2/\xi + pz - \frac{\varphi}{2}(p - p_0)^2.$$

The first-order condition is

$$1 + z - \frac{2}{\xi}p - \varphi(p - p_0) = 0, \tag{19}$$

which can be rearranged

$$\begin{aligned} 1 + z + \varphi p_0 &= \frac{2}{\xi}p^* + \varphi p^*, \\ 1 + z + \varphi p_0 &= p^*\left(\frac{2}{\xi} + \varphi\right), \end{aligned}$$

and solving for p, the optimal price is

$$\begin{aligned} p^* &= \frac{1 + z + \varphi p_0}{\frac{2}{\xi} + \varphi}, \\ &= \xi\frac{1 + z + \varphi p_0}{2 + \varphi\xi}. \end{aligned}$$

and total demand is

$$\begin{aligned} x &= 1 - \frac{1 + z + \varphi p_0}{2 + \varphi\xi}, \\ &= \frac{1 - z + \varphi(\xi - p_0)}{2 + \varphi\xi}. \end{aligned}$$

The Phillips curve is therefore given by

$$
\begin{aligned}
\kappa &= \frac{p_H - p_L}{x_H - x_L}, \\
&= \frac{\xi_H \frac{1+z+\varphi p_0}{2+\varphi \xi_H} - \xi_L \frac{1+z+\varphi p_0}{2+\varphi \xi_L}}{\left(1 - \frac{1+z+\varphi p_0}{2+\varphi \xi_H}\right) - \left(1 - \frac{1+z+\varphi p_0}{2+\varphi \xi_L}\right)}, \\
&= \frac{\frac{\xi_H}{2+\varphi \xi_H} - \frac{\xi_L}{2+\varphi \xi_L}}{\frac{1}{2+\varphi \xi_L} - \frac{1}{2+\varphi \xi_H}}, \\
&= \frac{2(\xi_H - \xi_L)}{\varphi \xi_H - \varphi \xi_L},
\end{aligned}
$$

which simplifies finally to

$$\kappa = \frac{2}{\varphi}$$

□

Proof of Lemma 3. The PC slope is given by

$$
\begin{aligned}
\kappa &= \frac{\bar{p} - p_0}{\bar{x} - \bar{x}_0}, \\
&= \frac{\varepsilon(\xi - \xi_0)(1+z)/2}{(1-\varepsilon)\left(\frac{p_0}{\xi_0} - \frac{p_0}{\xi}\right)}, \\
&= \frac{\varepsilon}{1-\varepsilon} \frac{\xi \xi_0 (1+z)}{2 p_0}.
\end{aligned}
$$

Since $p_0 = \xi_0(1+z)/2$ this simplifies to

$$\kappa = \frac{\varepsilon}{1-\varepsilon} \xi.$$

□

Proof of Lemma 4. Recall that $p_0 = \frac{\xi_0(1+z)}{2}$. Hence

$$
\begin{aligned}
k^*(\xi) &= \xi(1-z)^2/4 + p_0^2/\xi + z\xi - p_0(1+z), \\
&= \frac{\xi}{4}(1-z)^2 + \frac{\xi_0^2(1+z)^2}{4\xi} + z\xi - \frac{\xi_0(1+z)^2}{2}, \\
&= \frac{1}{4}\left(\xi(1-z)^2 + \frac{\xi_0^2(1+z)^2}{\xi}\right) - \frac{\xi_0(1+z)^2}{2} + z\xi.
\end{aligned}
$$

and we have

$$\frac{dk^*(\xi)}{d\xi} = \frac{(1-z)^2}{4} - \frac{1}{4}\frac{\xi_0^2(1+z)^2}{\xi^2} + z,$$
$$= \frac{1}{4}\left((1-z)^2 - \frac{\xi_0^2(1+z)^2}{\xi^2}\right) + z.$$

Thus, $k^*(\xi_0) = 0$ and is the global minimum. Then

$$\kappa = \frac{\bar{p} - p_0}{\bar{x} - x_0},$$

where, for $\xi = \xi_0$ the average price is p_0 and output is $x_0 = (1-z)/2$ since prices are already at the optimum. The average price is

$$\bar{p} = F(k^*(\xi))\frac{\xi(1+z)}{2} + (1 - F(k^*(\xi)))p_0,$$

and the level of output is

$$\bar{x} = F(k^*(\xi))\frac{1-z}{2} + (1 - F(k^*(\xi)))\left(1 - \frac{p_0}{\xi}\right).$$

Hence, we can write κ as

$$\kappa = \frac{F(k^*(\xi))\frac{\xi(1+z)}{2} + (1 - F(k^*(\xi)))p_0 - p_0}{F(k^*(\xi))\frac{1-z}{2} + (1 - F(k^*(\xi)))\left(1 - \frac{\xi_0}{2\xi}\right) - \frac{1-z}{2}}$$
$$= \frac{F(k^*(\xi))\left(\frac{\xi-\xi_0}{2}\right)(1+z)}{(1 - F(k^*(\xi)))\left(\frac{1+z-\xi_0}{2\xi}\right)}$$
$$= \frac{F(k^*(\xi))}{1 - F(k^*(\xi))}\left(\frac{\xi(\xi-\xi_0)(1+z)}{1+z-\xi_0}\right)$$

□

Proof of Lemma 5. First, k^* is still defined as above, but now we modify $p_0 = \frac{\xi_0(1+z_0)}{2}$. Hence we have

$$\begin{aligned}
k^*(z) &= \xi(1-z)^2/4 + p_0^2/\xi + z\xi - p_0(1+z), \\
&= \frac{\xi}{4}(1-z)^2 + \frac{\xi_0^2(1+z_0)^2}{4\xi} + z\xi - \frac{\xi_0(1+z_0)(1+z)}{2}, \\
&= \frac{1}{4}\left(\xi(1-z)^2 + \frac{\xi_0^2(1+z_0)^2}{\xi}\right) - \frac{\xi_0(1+z_0)(1+z)}{2} + z\xi.
\end{aligned}$$

If we want to consider a cost shock alone, we set $\xi = \xi_0$ to get

$$\begin{aligned}
k^*(z) &= \frac{1}{4}\left(\xi_0(1-z)^2 + \frac{\xi_0^2(1+z_0)^2}{\xi_0}\right) - \frac{\xi_0(1+z_0)(1+z)}{2} + z\xi_0, \\
&= \frac{1}{4}\left(\xi_0(1-z)^2 + \xi_0(1+z_0)^2\right) - \frac{\xi_0(1+z_0)(1+z)}{2} + z\xi_0, \\
&= \frac{\xi_0}{4}\left((1-z)^2 + (1+z_0)^2\right) - \frac{(1+z_0)(1+z)}{2} + z, \\
&= \frac{\xi_0}{4}\left((1-z)^2 + (1+z_0)^2 - 2(1+z_0)(1+z) + 4k\right), \\
&= \frac{\xi_0}{4}\left((1+z)^2 - 2(1+z_0)(1+z) + (1+z_0)^2\right), \\
&= \frac{\xi_0}{4}\left((1+z) - (1+z_0)\right)^2, \\
&= \frac{\xi_0}{4}\left(z_0 - z\right)^2.
\end{aligned}$$

□

A.2 Proofs for Section 4, Strategic Model with Demand Shocks

Proof of Proposition 1 . ONLY IF: To find the cutoff $\bar{\alpha}$ we need to confirm that in the low state the firm would rather charge the low flexible price than the high flexible price (which would fool the uninformed agents). If the flexible price $\{p_s\}$ is consistent with some PBE, then if the true state is L the firm will not prefer to deviate and offer the price p_H rather than the price p_L. Note that if the true state is L and the firm offers p_H, Insiders will know that the true state is L but Outsiders will believe the true state is H. Hence the firm will not want to offer $p_H = \xi_H(1+z)/2$ rather than $p_L = \xi_L(1+z)/2$ if and only if:

$$\begin{aligned}
\left(\frac{\xi_L(1+z)}{2} - z\xi_L\right)\left[1 - \frac{\xi_L(1+z)}{2}\frac{1}{\xi_L}\right] \geq \left(\frac{\xi_H(1+z)}{2} - z\xi_L\right)\Bigg\{\alpha\left[1 - \frac{\xi_H(1+z)}{2}\frac{1}{\xi_L}\right] \\
+(1-\alpha)\left[1 - \frac{\xi_H(1+z)}{2}\frac{1}{\xi_H}\right]\Bigg\}
\end{aligned}$$

Simplifying:

$$\xi_L \frac{(1-z)}{2}\left(\frac{1-z}{2}\right) \geq \left(\frac{\xi_H(1+z)-2\xi_L z}{2}\right)\left(\alpha\left(\frac{2-(1+z)\xi_H/\xi_L}{2}\right)+(1-\alpha)\left(\frac{1-z}{2}\right)\right)$$
$$\xi_L(1-z)(1-z) \geq (\xi_H(1+z)-2\xi_L z)(\alpha(2-(1+z)\xi_H/\xi_L)+(1-\alpha)(1-z))$$

Letting $\delta \equiv \xi_H/\xi_L$ and dividing both sides by ξ_L

$$(1-z)(1-z) \geq (\delta(1+z)-2z)(\alpha(2-(1+z)\delta)+(1-\alpha)(1-z))$$
$$(1-z)(1-z) \geq (\delta(1+z)-2z)((1-z)+\alpha(2-(1+z)\delta-(1-z)))$$
$$(1-z)(1-z) \geq (\delta(1+z)-2z)((1-z)+\alpha(1-(1+z)\delta+z))$$
$$(1-z)(1-z) \geq (\delta(1+z)-2z)((1-z)+\alpha(1+z)(1-\delta)).$$

Note that $\delta(1+z)-2z > 1-z$ and $\delta > 1$ so that $1-\delta < 0$. Then rearranging we have

$$\alpha(1+z)(\delta-1)(\delta(1+z)-2z) \geq (\delta(1+z)-2z)(1-z)-(1-z)(1-z)$$
$$\alpha(1+z)(\delta-1)(\delta(1+z)-2z) \geq (\delta(1+z)-2z-(1-z))(1-z)$$
$$\alpha(1+z)(\delta-1)(\delta+\delta k-2z) \geq (\delta-1+\delta k-z)(1-z)$$
$$\alpha(1+z)(\delta-1)(\delta+\delta k-2z) \geq (\delta-1)(1+z)(1-z)$$
$$\alpha \geq \frac{1-z}{\delta+\delta k-2z} \equiv \bar{\alpha},$$

which is the desired result. Note that if $z=0$, then we get $\bar{\alpha}=1/\delta$.

IF: Given that $\alpha \geq \bar{\alpha}$, We must construct a PBE in which prices along the equilibrium path are p_L, p_H. Hence we must show that when the true state is s the firm will not wish to deviate to a price $p \neq p_s$. PBE implies that when the Outsiders see the price p_s, they believe the true state is s, as in (a), (c). However, we are free to assign arbitrary beliefs to Outsiders if they see a price p different from both p_L and p_H, as in (b), (d); in that event we assign to Outsiders the belief that the true state is L. We must rule out four kinds of potentially profitable deviations

(a) The true state is L and the firm offers p_H.

(b) The true state is L and the firm offers $p \neq p_L, p_H$.

(c) The true state is H and the firm offers $p = p_H$.

(b) The true state is H and the firm offers $p \neq p_L, p_H$.

We have posited that when Outsiders see a price $p \neq p_L, p_H$ they believe the state is L and PBE guarantees that when Outsiders see the price p_L they believe the state is L, so we can subsume **(c), (d)** into

(e) The true state is L and the firm offers $p \neq p_H$.

We now verify **(a)** , **(b)** , and **(e)** in turn.

(a) This follows immediately by following the steps in the ONLY IF case above, but in reverse order, noting that each inequality is *equivalent* to the one above.

(b) We have posited that when Outsiders see a price $p \neq p_L, p_H$ they believe the state is L. Insiders know the true state so they also believe the state is L. Hence aggregate demand if the firm offers p will be $1 - p\frac{1}{\xi_L}$ and firm profit will be $(p - z\xi_L)[1 - p\frac{1}{\xi_L}]$. By definition, this quantity is maximized when $p = 1/2\frac{(1+z)}{\xi_L}$ and the maximum profit will be $1/4\frac{(1-z)^2}{\xi_L}$. However this is the profit when the firm offers p_L so this cannot be a profitable deviation for any such p.

(e) We must show that when the true state is H the firm's profit if it offers p_H is at least as great as when it offers $p \neq p_H$; i.e. we must show

$$\begin{aligned}\frac{\xi_H(1-z)^2}{4} &\geq (p - z\xi_H)\Big(\alpha[1 - p\frac{1}{\xi_H}] + (1-\alpha)[1 - p\frac{1}{\xi_L}]\\ &= \alpha(p - z\xi_H)[1 - p\frac{1}{\xi_H}] + (1-\alpha)(p - z\xi_H)[1 - p\frac{1}{\xi_L}] \qquad (20)\end{aligned}$$

By definition, $(p - z\xi_H)[1 - p\frac{1}{\xi_H}]$ would be maximized by setting $p = p_H$ and $(p - z\xi_H)[1 - p\frac{1}{\xi_L}]$ would be maximized by setting $p = p_L$ so we must certainly have

$$\alpha(p - z\xi_H)[1 - p\frac{1}{\xi_H}] \leq \alpha\left(\frac{\xi_H(1-z)^2}{4}\right) \qquad (21)$$

$$(1-\alpha)(p - z\xi_H)[1 - p\frac{1}{\xi_L}] \leq (1-\alpha)\left(\frac{\xi_L(1-z)^2}{4}\right) \qquad (22)$$

The result follows by adding the inequalities (21) and (22) together with $\xi_H > \xi_L$ so we have verified **(e)** .

Having verified **(a)** , **(b)** , and **(e)** , the proof is complete. □

Proof of Proposition 2 . Assume the sticky price $p_0 = \xi_0(1+z)/2$ is consistent with some PBE. Suppose that, in that PBE, the true state is L and the firm offers a price $p \neq p_0$. Because the Insiders know the true state, they will demand the quantity $1 - p\frac{1}{\xi_L}$. PBE requires that the Outsiders form some belief about the true state and demand a quantity that is optimal with respect to that belief about the true state; hence the Outsiders will demand $1 - p\mathbb{E}_o\left[\frac{1}{\xi}\right]$ where $\mathbb{E}_o\left[\frac{1}{\xi}\right]$ is their expectation of the shock. The profit of the firm will be:

$$d_L(p) = (p - z\xi_L)\Big(\alpha\big[1 - p\frac{1}{\xi_L}\big] + (1-\alpha)\big[1 - p\mathbb{E}_o\left[\frac{1}{\xi}\right]\big]\Big)$$

For every $p \neq p_0$, this expression will be minimized if the Outsiders assign probability 1 to the state L, in which case their expectation of the shock will be $E_o(\frac{1}{\xi_s}) = \frac{1}{\xi_L}$. Hence if the firm offers $p \neq p_0$ we must have

$$d_L(p) \geq (p - z\xi_L)[1 - p\frac{1}{\xi_L}]$$

In PBE the firm has no profitable deviation so it must be that $d_L(p_0) \geq d_L(p)$ for every p; in particular this inequality must hold when $p = p_L$. We conclude that

$$\left(\frac{\xi_0(1+z)}{2} - z\xi_L\right)\left(\alpha\left[1 - \frac{\xi_0(1+z)}{2}\frac{1}{\xi_L}\right] + (1-\alpha)\left[1 - \frac{\xi_0(1+z)}{2}\mathbb{E}_o\left[\frac{1}{\xi}\right]\right]\right) \geq \frac{\xi_L(1-z)^2}{4}$$

Because ξ_0 is the harmonic mean of ξ_L, ξ_H, $\mathbb{E}_o\left[\frac{1}{\xi}\right] = 1/\xi_0$; substituting and simplifying yields

$$(\xi_0(1+z) - 2z\xi_L)\left(\alpha\left[2 - \frac{\xi_0(1+z)}{\xi_L}\right] + (1-\alpha)(1-z)\right) \geq \xi_L(1-z)^2$$

The LHS is decreasing in α, and hence we must set α sufficiently low. Note that if $\alpha = 0$ then the LHS equals $(\xi_0(1+z) - 2z\xi_L)\,(1-z)$ and $\xi_0(1+z) - 2z\xi_L = \xi_0 + (\xi_0 - \xi_L)z - z\xi_L > \xi_L(1-z)$ since $\xi_0 > \xi_L$. Thus, there exists a threshold $\bar{\alpha}_0 > 0$ such that this inequality holds if $\alpha \leq \bar{\alpha}_0$.

Let $\delta_{0L} \equiv \frac{\xi_0}{\xi_L}$. Then we have

$$
\begin{aligned}
(\delta_{0L}(1+z)-2z)\left(\alpha\left[2-\delta_{0L}(1+z)\right]+(1-\alpha)(1-z)\right) &\geq (1-z)^2 \\
(\delta_{0L}(1+z)-2z)\left(1-z+\alpha\left[2-\delta_{0L}(1+z)-(1-z)\right]\right) &\geq (1-z)^2 \\
(\delta_{0L}(1+z)-2z)\left(1-z+\alpha\left[1+z-\delta_{0L}(1+z)\right]\right) &\geq (1-z)^2 \\
(1-z)-\alpha(\delta_{0L}-1)(1+z) &\geq \frac{(1-z)^2}{\delta_{0L}(1+z)-2z}
\end{aligned}
$$

Recall that $\delta_{0L} > 1$ since $\xi_L < \xi_0$, and therefore we have

$$
\begin{aligned}
(1-z)-\frac{(1-z)^2}{\delta_{0L}(1+z)-2z} &\geq \alpha(\delta_{0L}-1)(1+z) \\
\frac{(\delta_{0L}(1+z)-2z)(1-z)-(1-z)^2}{\delta_{0L}(1+z)-2z} &\geq \alpha(\delta_{0L}-1)(1+z) \\
\frac{(1-z)(\delta_{0L}(1+z)-2z-(1-z))}{\delta_{0L}(1+z)-2z} &\geq \alpha(\delta_{0L}-1)(1+z) \\
\frac{(1-z)(\delta_{0L}(1+z)-(1+z))}{\delta_{0L}(1+z)-2z} &\geq \alpha(\delta_{0L}-1)(1+z) \\
\frac{(1-z)(\delta_{0L}-1)(1+z)}{\delta_{0L}(1+z)-2z} &\geq \alpha(\delta_{0L}-1)(1+z)
\end{aligned}
$$

And therefore we can simplify to

$$
\alpha \leq \frac{1-z}{\delta_{0L}(1+z)-2z} \equiv \bar{\alpha}_0
$$

Recall that

$$
\bar{\alpha} \equiv \frac{1-z}{\delta(1+z)-2z} < \frac{1-z}{\delta_{0L}(1+z)-2z}
$$

since $\delta \equiv \xi_H/\xi_L > \delta_{0L} \equiv \xi_0/\xi_L$. Thus, if $\alpha < \bar{\alpha}$, it follows that $\alpha < \bar{\alpha}_0$ and therefore p_0 is a PBE.

To construct a PBE in which p_0 is offered in both states, we have to prescribe the behavior of Outsiders when a price $p \neq p_0$ is offered. As the argument above suggests, we posit that when when a price $p \neq p_0$ is offered, Outsiders believe the true state is L and hence demand $1 - p\frac{1}{\xi_L}$. Insiders know the true state s and demand $1 - p\frac{1}{\xi_s}$ so the profit of the firm is

$$
d_s(p) = (p - z\xi_s)\left(\alpha[1-p\frac{1}{\xi_s}] + (1-\alpha)[1-p\frac{1}{\xi_L}]\right) \tag{23}
$$

If the firm offers the putative equilibrium price p_0, the Outsiders' expectation of the future price will be $1/\xi_0$, so the profit of the firm will be

$$d_s(p_0) = \left(\frac{\xi_0(1+z)}{2} - z\xi_s\right)\left(\alpha\left[1 - \frac{\xi_0(1+z)}{2}\frac{1}{\xi_s}\right] + \frac{(1-\alpha)(1-z)}{2}\right)$$

The equilibrium condition is that

$$d_s(p_0) \geq d_s(p) \tag{24}$$

when $s = L$ and when $s = H$, under the assumption that $\alpha \leq \bar{\alpha}$. That the inequality (24) is satisfied when the true state $s = L$ follows from the exercise we just did. To see that (24) is satisfied when the true state $s = H$ is more complicated. First note that simplifying the right side of (23) yields

$$d_H(p) = (p - z\xi_H)\left(1 - p\left[\alpha\frac{1}{\xi_H} + (1-\alpha)\frac{1}{\xi_L}\right]\right)$$

Define $\xi_\alpha \equiv \left(\alpha\frac{1}{\xi_H} + (1-\alpha)\frac{1}{\xi_L}\right)^{-1}$ as the α-weighted harmonic mean of ξ_s. Since $\xi_\alpha < \xi_H$, we have

$$d_H(p) < (p - z\xi_\alpha)\left(1 - p\left[\alpha\frac{1}{\xi_H} + (1-\alpha)\frac{1}{\xi_L}\right]\right)$$

and the RHS is maximized by setting $p = \frac{\xi_\alpha(1+z)}{2}$ and equals $\frac{\xi_\alpha(1-z)^2}{4}$. Thus, it suffices to show that for $\alpha \leq \bar{\alpha}_0$

$$(p_0 - z\xi_H)\left(1 - p_0\left[\alpha\frac{1}{\xi_H} + (1-\alpha)\frac{1}{\xi_0}\right]\right) \geq \frac{\xi_\alpha(1-z)^2}{4}.$$

Note first that this is satisfied for $\alpha = 0$ but not for $\alpha = 1$; in the first case there are no Informed agents, so setting $p = p_0$ is strictly dominant; in the second case there are only Informed agents so the flexible price is optimal. Rearranging we have

$$(p_0 - z\xi_H)\left(1 - p_0\frac{1}{\xi_0} + p_0\alpha\left(\frac{1}{\xi_0} - \frac{1}{\xi_H}\right)\right) \geq \frac{(1-z)^2}{4\left(\frac{1}{\xi_L} - \alpha\left(\frac{1}{\xi_L} - \frac{1}{\xi_H}\right)\right)}.$$

Note that the LHS is increasing linearly in α since $\frac{1}{\xi_0} - \frac{1}{\xi_H} > 0$. The RHS is increasing with α. Multiplying we have

$$\left(\frac{1}{\xi_L} - \alpha\left(\frac{1}{\xi_L} - \frac{1}{\xi_H}\right)\right)(p_0 - z\xi_H)\left(1 - p_0\frac{1}{\xi_0} + p_0\alpha\left(\frac{1}{\xi_0} - \frac{1}{\xi_H}\right)\right) \geq \frac{(1-z)^2}{4},$$

which is a quadratic equation in α with a negative coefficient on α^2. Thus, if this holds at $\bar{\alpha}_0$ it holds for all $\alpha \leq \bar{\alpha}_0$. L'Huillier et al. (2022) verify this condition holds for $z = 0$. By continuity it holds for z sufficiently small. □

A.3 Proofs for Section 4.2, Strategic Model with Supply Shocks

Proof of Proposition 4. The proof is immediate. Note that conditional on the price p, Insiders and Outsiders have the same demand, $x = 1 - \frac{p}{\xi}$. Thus, profit maximization means choosing p to maximize $\left(1 - \frac{p}{\xi}\right)(p - z\xi)$, which yields the desired result. □

Proof of Proposition 5. Conditional on the price p, agents have the demand, $x = 1 - p\mathbb{E}\left[\frac{1}{\xi}\right]$. Hence, any incentive for firms to convey or hide information via p can only operate through the expectation on ξ. First, the firm can choose prices that communicates information about ξ, thus achieving full information. The full-information prices are given by

$$p_{H,H} = \frac{\xi_H(1 + z_H)}{2}, p_{H,L} = \frac{\xi_L(1 + z_H)}{2}, p_{L,H} = \frac{\xi_H(1 + z_L)}{2}, p_{L,L} = \frac{\xi_L(1 + z_L)}{2}.$$

Second, the firm can choose prices that do not communicate information about ξ. Since the demand shock is orthogonal to the shock to z_s, it is easy to show that the following prices maximize profits (as in the previous proof) without communicating information regarding ξ:

$$p_{0,H} = \frac{\xi_0(1 + z_H)}{2}, \quad p_{0,L} = \frac{\xi_0(1 + z_L)}{2}.$$

Hence, in the high cost state (z_H), the firm can choose a price that is sticky with respect to ξ by offering $p_{0,H} = \frac{\xi_0(1+z_H)}{2}$, the firm can choose a price that is flexible with respect to ξ by offering $p_{s,H} = \frac{\xi_s(1+z_H)}{2}$. The cutoff for choosing the sticky or flexible price is given by the threshold in equation (7). In this way, the firm can choose a price that is flexible with respect to both shocks by offering $p = \frac{\xi_s(1+z)}{2}$ or a price that is flexible with respect to the cost shock only by offering $p = \frac{\xi_0(1+z)}{2}$, which is sticky with respect to demand. The remaining results follow immediately from Proposition 3. □

A.4 Proof for Section 5

Proof of Proposition 6. Incentive-compatibility for the flexible-price equilibrium requires

$$\left(\frac{\xi_L(1-z)}{2}\right)\left(\frac{1-z}{2}\right) \geq \left(\frac{\xi_H(1+z)-2\xi_L z}{2}\right)\left(\alpha\left(1-\left(\frac{1+z}{2}\right)\frac{\xi_H}{\xi_L}\right)+(1-\alpha)\left(1-\left(\frac{1+z}{2}\right)\right)\right)$$

Simplifying:

$$\xi_L\frac{(1-z)}{2}\left(\frac{1-z}{2}\right) \geq \left(\frac{\xi_H(1+z)-2\xi_L z}{2}\right)\left(\alpha\left(\frac{2-(1+z)\xi_H/\xi_L}{2}\right)+(1-\alpha)\left(\frac{1-z}{2}\right)\right)$$

$$\xi_L(1-z)(1-z) \geq (\xi_H(1+z)-2\xi_L z)\left(\alpha\left(2-(1+z)\xi_H/\xi_L\right)+(1-\alpha)(1-z)\right)$$

Letting $\delta = \xi_H/\xi_L$ and dividing both sides by ξ_L

$$(1-z)(1-z) \geq (\delta(1+z)-2z)\left(\alpha\left(2-(1+z)\delta\right)+(1-\alpha)(1-z)\right)$$

$$(1-z)(1-z) \geq (\delta(1+z)-2z)\left((1-z)+\alpha(2-(1+z)\delta-(1-z))\right)$$

$$(1-z)(1-z) \geq (\delta(1+z)-2z)\left((1-z)+\alpha(1-(1+z)\delta+z)\right)$$

$$(1-z)(1-z) \geq (\delta(1+z)-2z)\left((1-z)+\alpha(1+z)(1-\delta)\right)$$

Note that $\delta(1+z)-2z > 1-z$ and $\delta > 1$ so that $1-\delta < 0$. Then rearranging we have

$$\alpha(1+z)(\delta-1)(\delta(1+z)-2z) \geq (\delta(1+z)-2z)(1-z)-(1-z)(1-z)$$

$$\alpha(1+z)(\delta-1)(\delta(1+z)-2z) \geq (\delta(1+z)-2z-(1-z))(1-z)$$

$$\alpha(1+z)(\delta-1)(\delta+\delta k-2z) \geq (\delta-1+\delta k-z)(1-z)$$

$$\alpha(1+z)(\delta-1)(\delta+\delta k-2z) \geq (\delta-1)(1+z)(1-z)$$

$$\alpha \geq \frac{1-z}{\delta+\delta k-2z} = \bar{\alpha}$$

Inverting the requirement (if z varies), then the cutoff for marginal cost solves

$$\alpha(\delta+\delta k-2z) \geq 1-z$$

$$\alpha\delta+\alpha(\delta-2)k \geq 1-z$$

$$(\alpha(\delta-2)+1)k \geq 1-\alpha\delta$$

Note that $1 + \alpha\delta > 1 + \alpha > 2\alpha$. Hence, $\alpha(\delta - 2) + 1 > 0$, and so we have

$$z \geq \frac{1 - \alpha\delta}{\alpha(\delta - 2) + 1} = \bar{z}$$

□

B General Equilibrium Framework

This section lays down a general equilibrium framework in which our model of price stickiness can be embedded. We have two goals. The first is to clarify that the earlier results can be obtained in a model with labor supply (and no endowments). The second is to clarify that the earlier results can be obtained in a model with where money plays an essential role. The setup's pieces are standard. However, putting the pieces together is quite involved. Therefore we start its description with a preview. We subsequently fully describe every piece of the model.

B.1 Preview

The setup is based on the foundational papers by Lagos and Wright (2005) and Lucas and Stokey (1987). As Lagos and Wright (2005), we exploit quasilinearity and periods that are divided in a day and a night to be able to handle agent (informational) heterogeneity. As Lucas and Stokey (1987), we use a cash-in-advance model with credit and cash goods. The presence of credit goods is key for specifying trading in goods markets with partially informed consumers.

The population of the economy is composed by a unit mass of households. These households own a unit mass of firms, which operate in different and segmented geographic locations called islands. There is a unit mass of islands, and on each island there is a single firm.

Households are divided into workers and consumers. Workers supply labor; consumers shop for consumption goods.

As in Lagos and Wright (2005), each period is divided into two subperiods: a day and a night. All the action of interest takes place during the day; the night is simply introduced as a technical device to close the model. Trading of credit goods takes place during the day; trading of cash goods takes place during the night.

The exogenous aggregate state of the economy is given by a preference shock θ, which is the discount factor between the day and the night. As in most of the literature, this preference shock is a modeling device to generate fluctuations in nominal aggregate demand. As in the simple model presented in previous sections' of the paper, a key assumption of our setup is that there is household heterogeneity in the information about this aggregate shock. Some households may be imperfectly informed about the value of discount factor at night.[17] We model this by making the sharp assumption that a fraction of households is perfectly informed about the realization of the shock and the complement is uninformed about the realization of the shock.

Firms, by assumption, are informed about the preference shock. We motivate this simplifying assumption by a story in which firms are able to aggregate consumer demand via goods market trading. So long as a non-zero mass of each firm's consumers are informed, their demand then reveals the aggregate preference shock to firms. To simplify the exposition, here we simply assume that firms are informed right from the start. On the other hand, imperfectly informed consumers learn by looking at firms' prices.[18] In fact, firms and consumers play a sequential game. Consumers and firms meet in decentralized locations. Each firm posts a price, consumers observe the price, and then post their demand.

The central bank controls money supply, which determines relative price between the night and day. The central bank uses a rule to determines its policy. This rule depends on deviations of inflation from a target and on the output gap.

B.2 Full Model

Population and Geography. The economy is populated by households, firms, and a central bank (CB). The geography is given by a unit mass of islands, and a mainland. Each island is populated by a continuum of households of mass one and is served by a single monopolistic firm. The mainland is visited by all consumers in the economy at given dates, and is served by a competitive representative firm. Households are divided in workers and consumers.

Time Structure. Time is discreet. Similar to Lagos and Wright (2005), periods are divided into two subperiods, called "day" and "night". Following their notation, we will denote day variables in lower case, and night variables in upper case. Subperiods are indexed by t: $t = 0$ signifies the day, and $t = 1$ signifies the night. (However, to simplify the notation, we skip t notation when possible.) Periods are indexed by τ and run from $\tau = 0$ to infinity.

[17] One can also think about this shock representing a shift in marginal utility at night. Under this interpretation, the assumption is that, during the day, imperfectly informed households do not receive full information about marginal utility at night.

[18] Notice that, our informational assumptions force us to move away from monopolistic competition (or other forms of centralized goods markets).

Goods Markets. We start by describing day-time trading in the decentralized market. Each mass of consumers are served by a price-setting monopolist (on a given island), which sells good c at a nominal price p. These decentralized goods are bought on credit.

We now describe the functioning of the night-time, centralized, market. At night, all consumers are sent to the mainland. There, they consume an aggregate good C, produced by a competitive firm, and sold at an aggregate nominal price P. We also refer to this aggregate price P as the night price level. This good is sold in exchange for cash.

Labor Markets. During both the day and the night labor markets are open. During the day, workers supply labor in a centralized labor market. Local firms hire workers from this centralized market. At night, labor is supplied in the mainland. Both (day and night) markets are competitive. Daytime labor is denoted l; nighttime labor is denoted L. We denote wages as w and W, respectively.

Credit, Financial, and Money Markets. During the day, all transactions take place on credit. Consumers buy consumption goods on credit, workers bring back wages, and firms pay profits (the firm is owned by local households).

At night, goods are bought in cash. (Labor is supplied on credit.)

The money market opens only at (the end of the) night. Similar to Lucas and Stokey (1987), all credit transactions are settled at this moment. A (long-term) bond is available across periods. These are trades in exchange of money holdings for the next period $\tau + 1$. Long term bonds and cash holdings are denoted B and M respectively.

Exogenous Aggregate State. The exogenous aggregate state is given by the realization of a preference shock θ_τ. We specify the process for the preference shock below.

Central Bank. The central bank sets the money supply following the commitment a rule, which we specify below. The money supply determines the price level during the night P.

Information Structure. Consumers are heterogeneous in terms of the information they possess. There are two types of consumers: Insiders (informed consumers) $\iota \in I$ and Outsiders (uninformed consumers) $o \in O$. Insiders are perfectly informed about the state θ_τ; Outsiders are uninformed about the state but know the probability distribution, and may draw inferences from the price set by the firm with which they trade. The fraction $\alpha \in [0, 1)$ of Insiders on a particular island varies across islands. We assume the distribution of α is given by a cdf F whose support is not a singleton and has the property that

$$\lim_{\alpha \to 1} F(\alpha) = 1$$

That is, the fraction of islands on which all consumers are Insiders is 0. All other agents in the economy have perfect information.

All of the above is common knowledge.

Household Optimization. We start by presenting an inner problem of the household. In this problem, household solve for all variables that trade in credit. This is the "day-to-night" problem where the action happens. (The outer problem is presented below.)

We index a typical household by j. The inner problem at date τ consists in solving

$$\max_{c_\tau, l_\tau, L_\tau} \; \mathbb{E}_{j\tau} \left[\Big(u(c_\tau) - l_\tau \Big) + \theta_\tau \Big(U(\bar{C}) - L_\tau \Big) \right]$$

where choices variables have been defined above. The variable $\bar{C}$ denotes a fixed allocation of the nighttime consumption good. Since this good is traded in cash, its consumption is fixed in the inner problem (the outer problem will determine this quantity). The random variable θ_τ is the exogenous aggregate state, which determines the discount factor between the day and the night. Following the previous sections, we specify the process for θ_τ to be very simple, by assuming it follows an i.i.d. binary Markov chain with two values, θ_L and θ_H, with $Pr(\theta_L) = Pr(\theta_H) = 1/2$ and $\mathbb{E}[\theta_\tau] = \theta < 1$. The realization θ_H corresponds to the high state, and the realization θ_L corresponds to the low state. The household values daytime consumption relatively more in the high state (and hence demand is higher than in the low state). Hence, the realizations are such that $\theta_L > \theta_H$.[19] The utility functions $u(\cdot)$ and $U(\cdot)$ are assumed to be twice continuously differentiable on $\mathbb{R}^{++}$, strictly increasing, and strictly concave. Below, we make an assumption on $u(\cdot)$ such that the monopolist's problem has a solution.[20] We assume that there is a value of C such that $U'(C) = 1/\beta$. The expectation operator is indexed by j to signify the household member's information set at the time they make a choice.

[19] The model allows for richer specifications of the exogenous process for the state, such as persistent Markov chains and AR(1). (To simplify the notation of this GE framework, we omit the subscript s to denote the state as in the previous sections, and simply use the notation θ_τ.)

[20] The earlier sections assume quadratic utility, which is a convenient assumption for welfare calculations. Here in the GE framework we aim to show that this particular restriction is not needed to find a general equilibrium solution.

This problem is subject to a constraint given by

$$p_\tau c_\tau + P_\tau \bar{C} = \pi_\tau + w_\tau l_\tau + W_\tau L_\tau \tag{25}$$

where prices have been defined above and π_τ are profits.

Denoting by λ_τ the Lagrange multiplier of the constraint (25), the first-order condition for daytime consumption c is

$$u'(c_\tau) = \mathbb{E}_{j\tau}\left[\lambda_\tau p_\tau\right]$$

It is important to emphasize that, depending on the index j, this condition may be taken under imperfect information. Indeed, it defines the choice, in a decentralized market, by a consumer that can be an Outsider. (Insiders have full information about the product $\lambda_\tau p_\tau$.) Notice however that Outsiders observe the price of the firm they meet, p_τ, and hence the expectation conditional on this price. Therefore, it can be taken out of the expectation operator.

We obtain the following set of first-order conditions:

$$u'(c_\tau) = p_\tau \mathbb{E}_{j\tau}\left[\lambda_\tau\right]$$
$$1 = \lambda_\tau w_\tau$$
$$\theta_\tau = \lambda_\tau W_\tau$$

(The remaining optimality conditions are taken under perfect information, since they involve choices in centralized markets.)

From the second equation we observe that $\lambda_\tau = 1/w_\tau$. Further manipulating the equations above we can summarize the set of first-order conditions to

$$u'(c_\tau) = p_\tau \mathbb{E}_{j\tau}\left[\frac{\theta_\tau}{W_\tau}\right] \tag{26}$$
$$\frac{1}{w_\tau} = \theta_\tau \frac{1}{W_\tau} \tag{27}$$

We continue by presenting the outer problem of the household. This is the problem solved from one day to the other. (Below we will formally establish the relation between both problems.) This problem will give rise to an explicit role for money and hence allows us to define the monetary policy instrument.

Define

$$\mathcal{U}(c_\tau, l_\tau, C_\tau, L_\tau) = \Big(u(c_\tau) - l_\tau\Big) + \theta_\tau \Big(U(C_\tau) - L_\tau\Big)$$

In the outer problem, the household needs to solve

$$\max_{c_\tau, l_\tau, C_\tau, L_\tau, M_\tau, B_\tau} \; \mathbb{E}_{j\tau} \left[\sum_{\tau=0}^{\infty} \beta^\tau \, \mathcal{U}(c_\tau, l_\tau, C_\tau, L_\tau) \right]$$

which involves choosing infinite sequences of consumption, labor supply, money and bond holdings subject to

$$p_\tau c_\tau + P_\tau C_\tau + B_\tau + M_\tau = w_\tau l_\tau + W_\tau L_\tau + M_{\tau-1} + T_\tau + \left(1 + i_\tau^{LT}\right) B_{j\tau-1} + \pi_\tau \tag{28}$$

where i_τ^{LT} is a long-term nominal interest rate, and T_τ is a lump-sum cash transfer set by the central bank. Purchases of the cash good are also subject to a cash-in-advance (CIA) constraint

$$P_\tau C_\tau \leq M_{\tau-1} + T_\tau \tag{29}$$

Denoting by χ_τ the multiplier on the budget constraint (28) and by ψ_τ the multiplier on the CIA constraint (29), we get the set of first-order conditions

$$\beta^\tau u'(c_\tau) = \mathbb{E}_{j\tau} \left[\chi_\tau p_\tau \right] \tag{30}$$

$$\beta^\tau = \chi_\tau w_\tau \tag{31}$$

$$\beta^\tau \theta_\tau U'(C_\tau) = (\chi_\tau + \psi_\tau) P_\tau \tag{32}$$

$$\beta^\tau \theta_\tau = \chi_\tau W_\tau \tag{33}$$

$$\chi_\tau = \mathbb{E}_\tau \left[\chi_{\tau+1} + \psi_{\tau+1} \right] \tag{34}$$

$$\chi_\tau = \left(1 + i_{\tau+1}^{LT}\right) \mathbb{E}_\tau \left[\chi_{\tau+1} \right] \tag{35}$$

where it is important to notice the presence of two different expectation operators, the daytime expectation operator $\mathbb{E}_{j\tau}[\,\cdot\,]$ (conditional on consumer j's information), and the nighttime expectation operator $\mathbb{E}_\tau[\,\cdot\,]$ (conditional on full information, which is available in the centralized market).

From (31) and (33), we observe that $\chi_\tau = \beta^\tau / w_\tau$ and $\chi_\tau = \beta^\tau \theta_\tau / W_\tau$. Thus,

$$\frac{1}{w_\tau} = \theta_\tau \frac{1}{W_\tau}$$

which is the same as (27). Also, plugging in the expression for χ_τ obtained from (33) into (30), we get

$$u'(c_\tau) = p_\tau \mathbb{E}_{j\tau}\left[\frac{\theta_\tau}{W_\tau}\right] \tag{36}$$

which is the same as (26).

The remaining conditions (determining the demand for money and bonds) can be simplified as follows. Equation (32), one period forward, is $\beta^{\tau+1}\theta_{\tau+1}U'(C_{\tau+1}) = (\chi_{\tau+1} + \psi_{\tau+1})P_{\tau+1}$. Solving for $\chi_{\tau+1} + \psi_{\tau+1}$, and using the expression for χ_τ, equation (34) becomes

$$\frac{\theta_\tau}{W_\tau} = \beta\ \mathbb{E}_\tau\left[\frac{\theta_{\tau+1}}{P_{\tau+1}}U'(C_{\tau+1})\right] \tag{37}$$

Finally, equation (35) is equivalent to

$$\frac{\theta_\tau}{W_\tau} = \beta\left(1 + i^{LT}_{\tau+1}\right)\mathbb{E}_\tau\left[\frac{\theta_{\tau+1}}{W_{\tau+1}}\right] \tag{38}$$

Production. All firms in the economy have a linear technology and produce using only labor. Within every period, monopolist of the decentralized market produces c according to the production function $c_\tau = Al_\tau$. For simplicity, we assume that $z \equiv 1/A$ is commonly known.

The competitive firm produces C according to the production function $C_\tau = L_\tau$, where productivity has been normalized to 1.

Game in the Decentralized Market. The equilibrium notion for the game played between consumers and firms is the one described in full detail in the body. Below we shall prove that, this setup is tractable in the following sense: Any equilibrium of this game is part of a general equilibrium for the whole economy.

Central Bank. The central bank sets the money supply M^S_τ. An increase of the money supply (away from its steady state value) is expansionary since it is increases aggregate demand, and vice versa. The central banks behaves by adjusting money supply as a function of inflation and the output gap, according to the following rule:

$$M^S_\tau = M_0\left(\hat{p}_\tau\right)^{-\phi_\pi}\left(\hat{x}_\tau\right)^{-\phi_x} \tag{39}$$

where M_0 is the natural level of the money supply, $\hat{p}_\tau$ is inflation, defined as the percentage deviation of the price level P_τ away from steady state p_0: $\hat{p}_\tau = \int p(\alpha)dF(\alpha)/p_0$, and $\hat{x}_\tau$ is the output gap, defined as the percentage deviation of aggregate output from steady state $y_0 = 1/2$: $\hat{x}_\tau = \int y(\alpha)dF(\alpha)/(1/2)$. Below we show that this policy rule can be expressed as a rule for a short-term interest rate rate.

We are finally in a position where we can define a general equilibrium for the economy.

Definition of Equilibrium. A (general) equilibrium of this economy is given by consumption allocations, labor supply, bond holdings and money demand (for each household) $\{c_\tau, C_\tau, l_\tau, L_\tau, B_\tau, M_\tau\}$, labor demand (for each firm) $\{l^D_{j\tau}, L^D_\tau\}$, profits $\{\pi_\tau\}$, money supply $\{M^S_\tau\}$, nominal transfers $\{T_\tau\}$, nominal prices $\{p_\tau, P_\tau\}$, nominal wages $\{w_\tau, W_\tau\}$, long-term nominal interest rates $\{1 + i^{LT}_\tau\}$, for all τ, such that:

1. Households' conditions for optimality and corresponding constraints are satisfied;
2. The price-setting game is solved as specified above;
3. The representative firm maximizes profits taking the price as given;
4. The CB sets money supply as specified by the rule above;
5. Goods, labor, bonds, and money markets clear.

General Equilibrium Characterization. First, we conjecture that C_τ is constant in equilibrium. If so, then the price of this good is pinned down by the cash in advance constraint. We denote this constant $C_\tau = \bar{C}$. Second, we conjecture that $M_\tau = M^S_\tau$, for all τ. Then, $P_\tau = M_\tau/\bar{C}$.

By the optimality condition for the production of the representative firm, the nominal wage $W_\tau = P_\tau$ (since productivity is normalized to 1). Thus,

$$P_\tau = W_\tau = \frac{M_\tau}{\bar{C}} \tag{40}$$

Now, taking equation (37) and writing it as

$$\frac{\theta_\tau}{M_\tau} = \beta U'(\bar{C})\ \mathbb{E}_\tau\left[\frac{\theta_{\tau+1}}{M_{\tau+1}}\right]$$

reveals that as long as $U'(\bar{C}) = 1/\beta$ and θ_τ/M_τ is a martingale, equation (37) is satisfied. A monotonic rule can be mapped into a degree of monetary policy adjustment γ. Hence, we write the rule

$$\frac{1}{M_\tau} = \gamma \frac{1}{\theta_\tau} + (1-\gamma)\frac{1}{M_0}$$

According to this rule, when $\gamma = 1$, there is full adjustment ($M_\tau = \theta_\tau$), and when $\gamma = 0$, there is no adjustment ($M_\tau = M_0$). This rule can be written

$$\frac{\theta_\tau}{M_\tau} = \gamma + (1-\gamma)\frac{\theta_\tau}{M_0}$$

Taking the expectation of θ_τ / M_t shows, trivially, that this ratio is a martingale, and thus equation (37) is satisfied.

A similar argument for $i_\tau^{LT} = 1/\beta - 1$ shows that equation (38) is satisfied.

Since this is a closed economy with a zero net supply of bonds, we can simply set $B_\tau = 0$ for all households. It remains to check that the labor markets clear. The centralized market clears when each household supplies $L_\tau = C_\tau$. In the decentralized market, each household's labor supply is set to satisfy their respective budget constraints. Aggregating the budget constraint gives the economy's resource constraint, and from this one can establish that the labor market clears in each island. (Notice, this implies that any equilibrium solution to the game played between firms and consumers is a GE. This ensure a tractable and isolated treatment of the game.)

Finally, set $T_\tau = M_\tau - M_{\tau-1}$. At this point, we are able to verify our money demand and centralized good consumption conjectures. This completes the characterization of the GE framework.

Equivalence Results. In order to understand the sense in which the program of the household admits an inner and an outer problem, notice first that the first order condition for c_τ in both problems are the same (equations (26) and (36)). Also, since the equilibrium in the outer problem requires $L_\tau = C_\tau$, and since $W_\tau = P_\tau$, $M_\tau = M_{\tau-1} + T_\tau$ and $B_\tau = 0$, then, setting $E = \bar{C}$ the budget constraints in both problems reduce to

$$p_\tau c_\tau = \pi_\tau + w_\tau l_\tau$$

leading to the same choice of l_τ in both problems. The following result has then just been established.

Lemma 6. *The equilibrium allocations of c_τ, l_τ, L_τ in the inner problem are the same as in the outer problem. Moreover, the equilibrium allocation C_τ is an admissible endowment E of the inner* *problem.*

To obtain the simple, partial equilibrium, model in section 3, interpret the cash good as a numeraire good. Since the credit good and the cash good are purchased in subsequent periods (call them period 0 and period 1), the price P_τ can be interpreted as the price of an asset traded at period 0, that pays 1 unit of the numeraire good in period 1. Denote this price Q_τ. Then, $Q_\tau = P_\tau$. Moreover, since in this simple model marginal costs are zero, take the limit $A \to \infty$, which implies zero labor demand.

In the simple model, the choice of C is determined by the budget constraint. Since households are heterogeneous in terms of their information and implied choice of c, nothing guarantees that C is equal to E. However, by the linearity, one can verify that the aggregate quantity of C is indeed equal to E.

Notice that $M_\tau = E \cdot Q_\tau$. So the rule (39) is

$$E \cdot Q_\tau = E \cdot Q_0 \left(\hat{p}_\tau\right)^{-\phi_\pi} \left(\hat{x}_\tau\right)^{-\phi_x}$$

which is

$$1 + i_\tau = (1 + i_0) \left(\hat{p}_\tau\right)^{\phi_\pi} \left(\hat{x}_\tau\right)^{\phi_x}$$

In logs

$$\log(1 + i_\tau) = \log(1 + i_0) + \phi_\pi \hat{p}_\tau + \phi_x \hat{x}_\tau$$

Finally, the simple model can be written using two periods only, which allows to drop the τ index and keep only the lower case and upper case notation for $t = 0$ and $t = 1$.

Thereby, the following lemma establishing the alleged equivalence has been proven.

Lemma 7. *The model presented in section 3 has the same equilibrium allocation of c_τ as the full GE model. Also, the aggregate consumption of C in the simple model is equal to $E = \bar{C}$.*

Finally, we note it is straightforward to extend this result, based on the full GE model, to the case of positive marginal costs with finite A and $z = 1/A$.